Sólido deformable: cables

Mariano Vázquez Espí

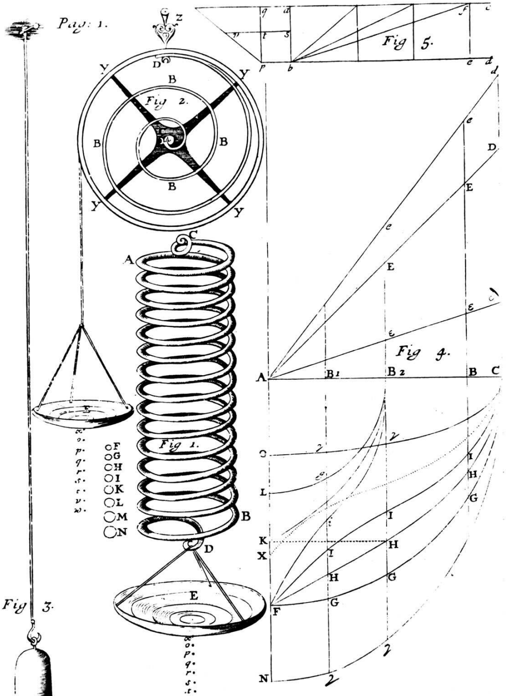
Madrid (España), 1 de octubre de 2010.

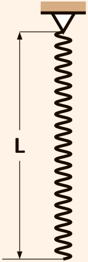
Robert Hooke (1635–1703)
—Físico, astrónomo y naturalista

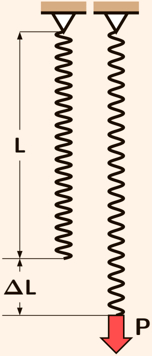
Entre otras cosas, introdujo el concepto de *célula* y analizó la anatomía de los insectos.

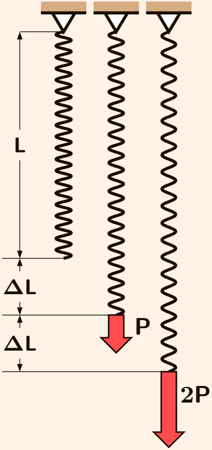
Thomas Young (1773–1829)
—Físico y médico

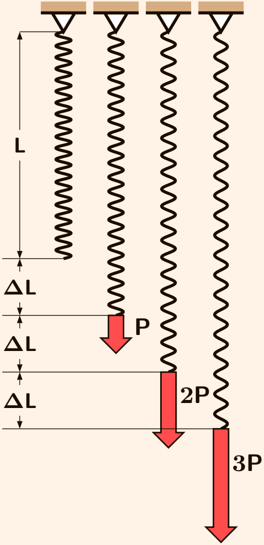
Entre otras cosas, introdujo el concepto moderno de *energía* y contribuyó a descifrar la escritura jeroglífica egipcia.

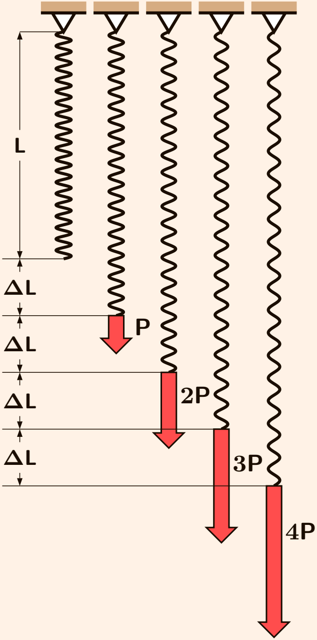



L

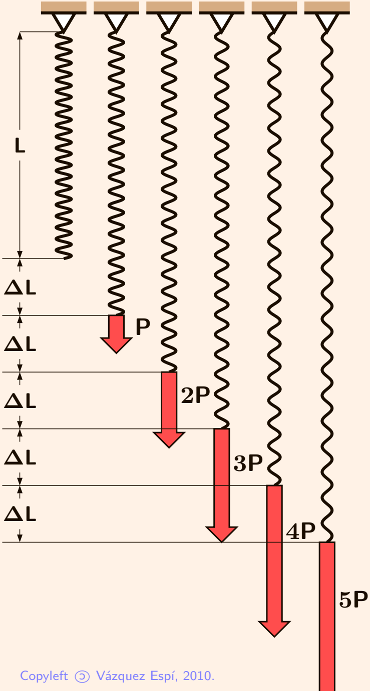

L
ΔL
P

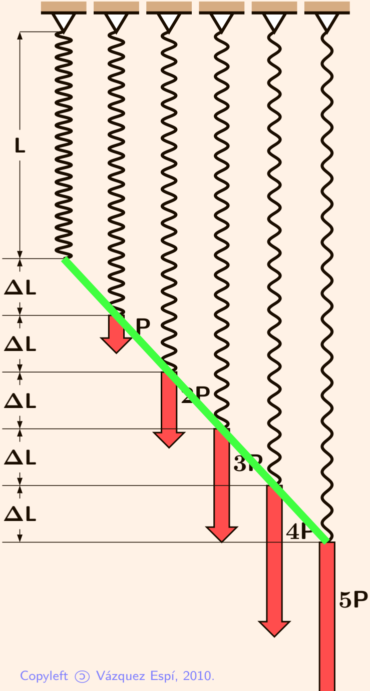

L
ΔL
ΔL
P
2P

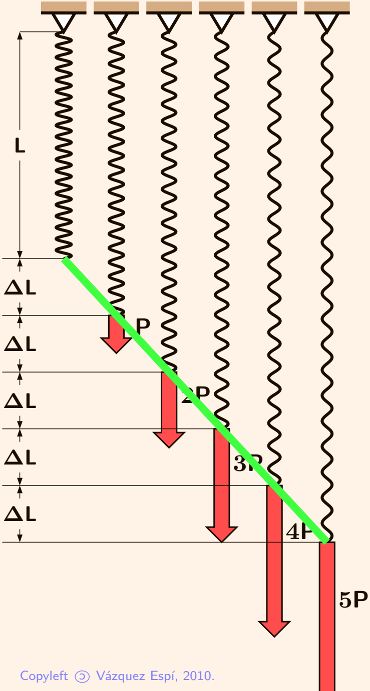

L
ΔL
ΔL
ΔL
P
2P
3P

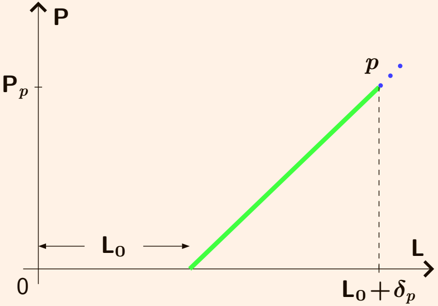


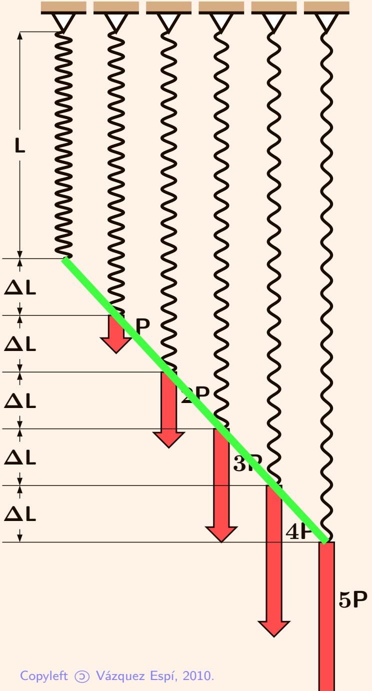
L
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P


L
ΔL
P
ΔL
2P
ΔL
3P
ΔL
4P
ΔL
5P

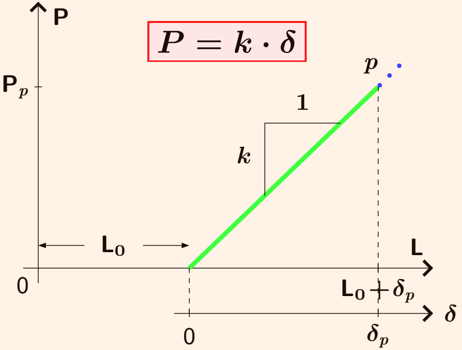

L
ΔL
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P
5P

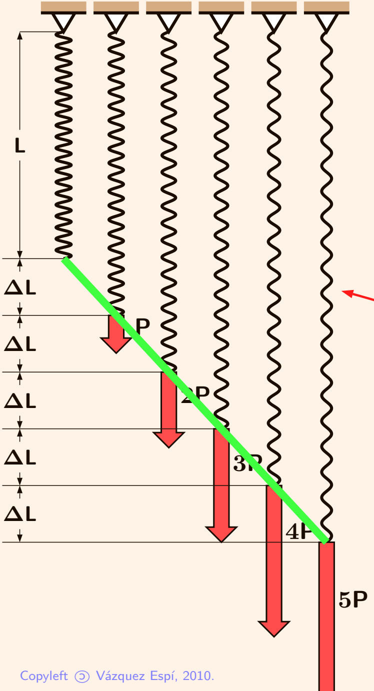

L
ΔL
ΔL
ΔL
ΔL
ΔL
P
2P
3P
4P
5P

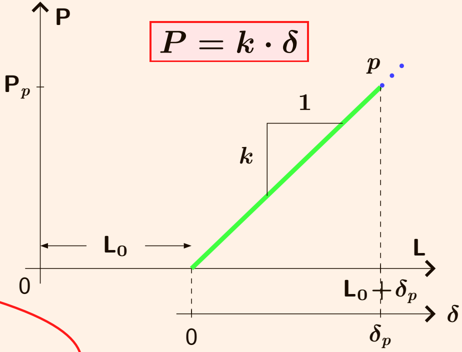

P
p
P_p
L_0
L
0
$L_0 + \delta_p$



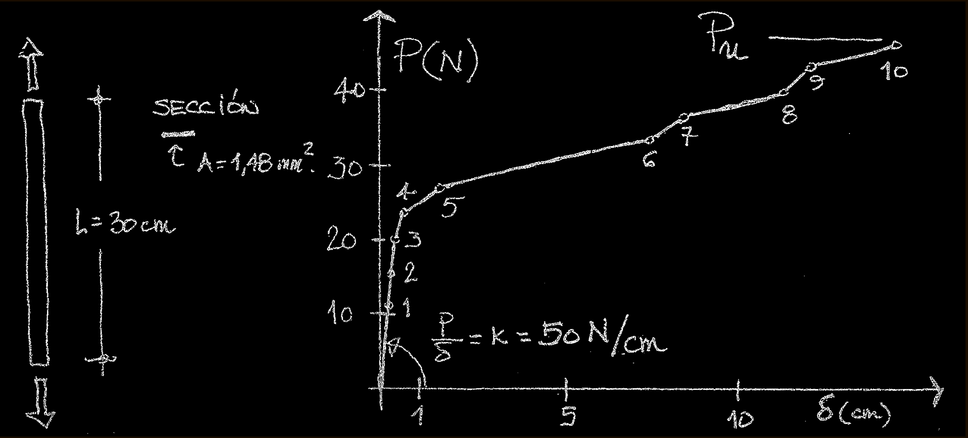
$$P = k \cdot \delta$$

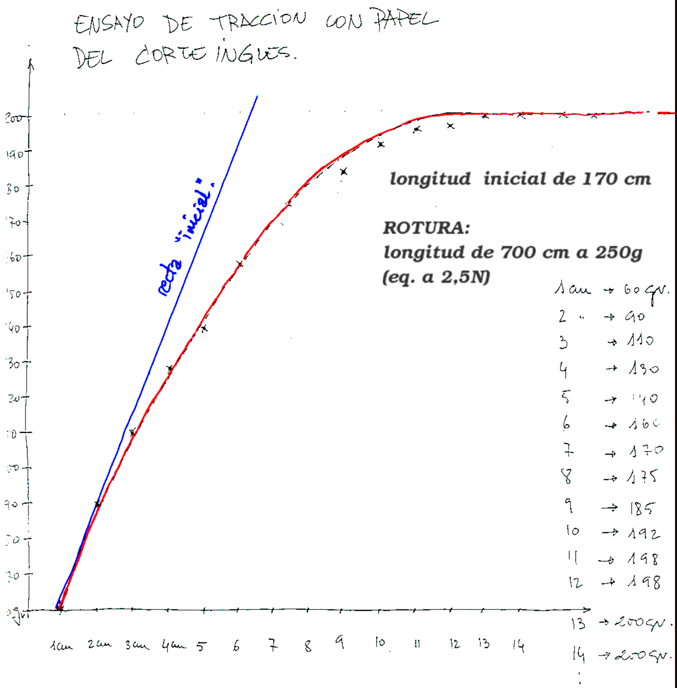


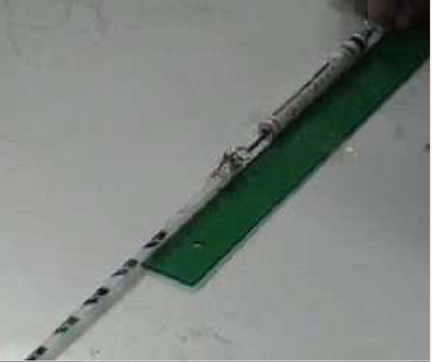


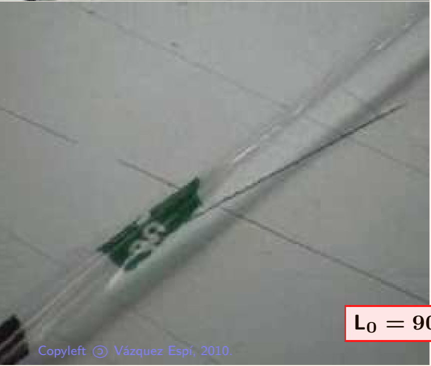
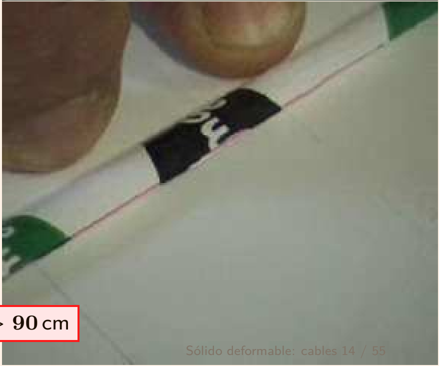


¡el muelle se estrecha!

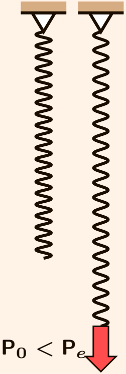

P(N)
Pu
SECCIÓN
A=1,48 mm².
L=30 cm
40
30
20
10
1
2
3
4
5
6
7
8
9
10
P/δ = k = 50 N/cm
1
5
10
δ (cm)

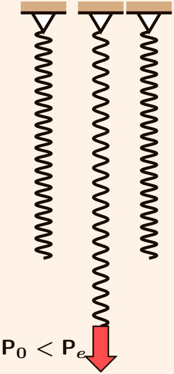

ENSAYO DE TRACCIÓN CON PAPEL
DEL CORTE INGLÉS.
recta "inicial".
longitud inicial de 170 cm
ROTURA:
longitud de 700 cm a 250g
(eq. a 2,5N)
1 cm → 60 gr.
2 " → 90
3 → 110
4 → 130
5 → 140
6 → 160
7 → 170
8 → 175
9 → 185
10 → 192
11 → 198
12 → 198
13 → 200 gr.
14 → 200 gr.
1cm 2cm 3cm 4cm 5 6 7 8 9 10 11 12 13 14

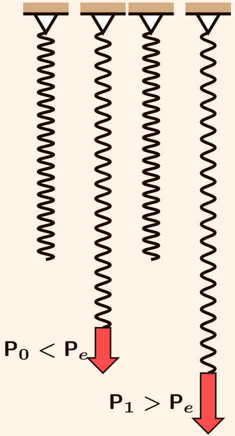


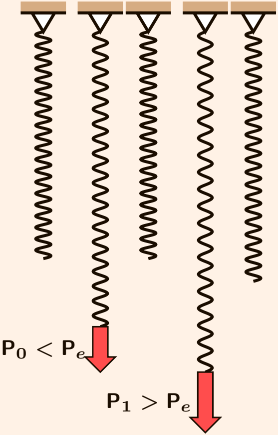



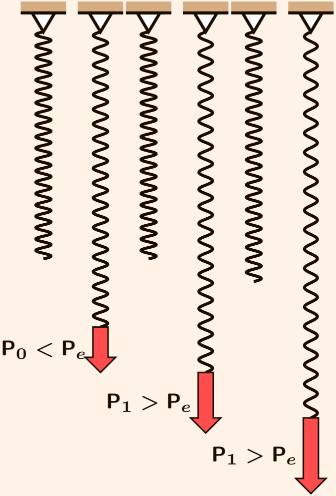
$\mathbf{L_0 = 90\,cm,\ \delta_u > 90\,cm}$


$\mathbf{P}_0 < \mathbf{P}_e$

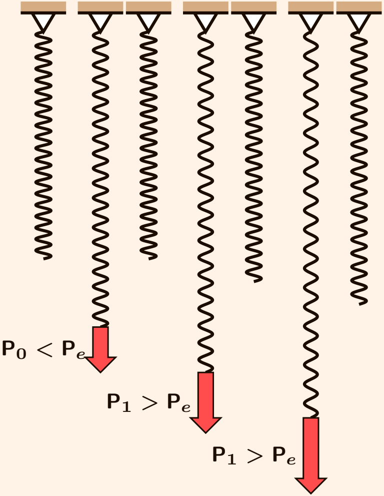

$\mathbf{P_0} < \mathbf{P}_e$

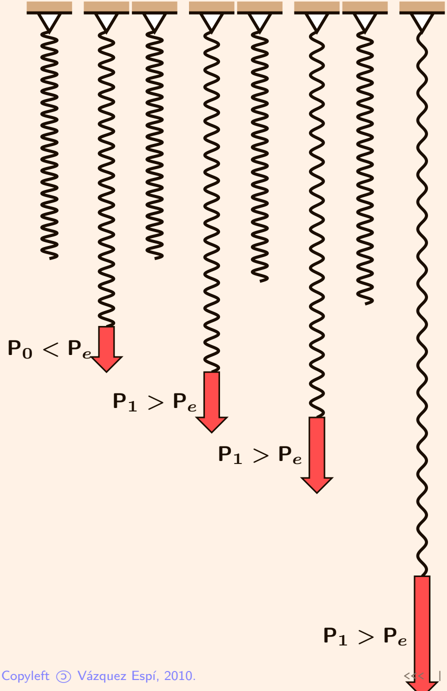

$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$

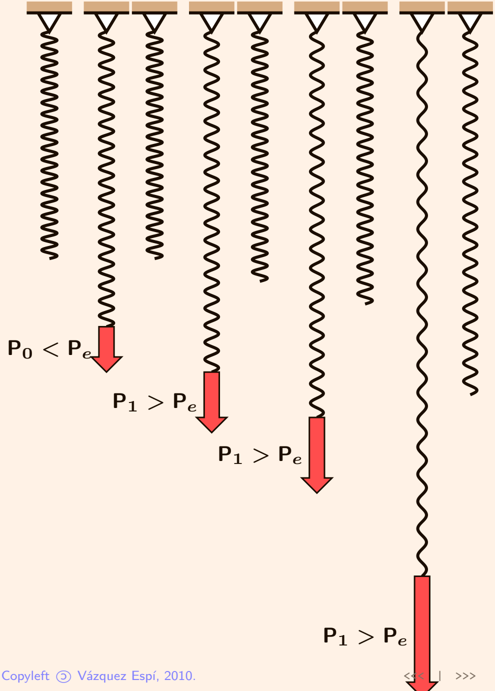

$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$



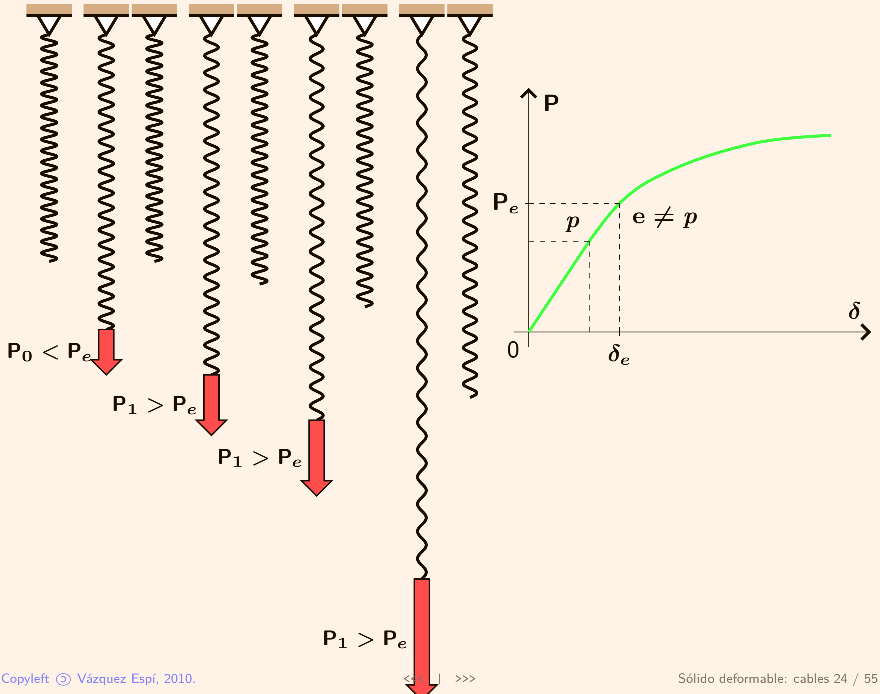
$\mathbf{P}_0 < \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$
$\mathbf{P}_1 > \mathbf{P}_e$


P0 < Pe
P1 > Pe
P1 > Pe


$P_0 < P_e$
$P_1 > P_e$
$P_1 > P_e$
$P_1 > P_e$

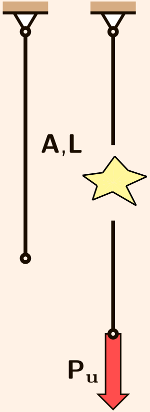


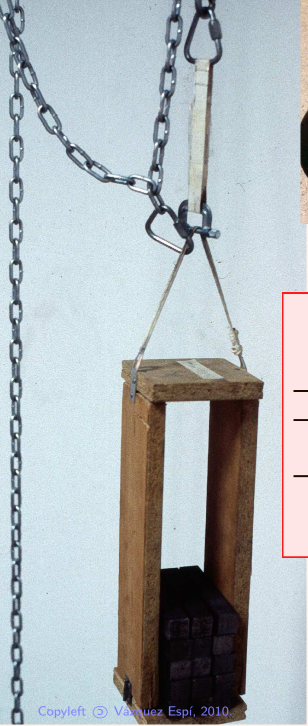
$\mathbf{P_0} < \mathbf{P}_e$
$\mathbf{P_1} > \mathbf{P}_e$
$\mathbf{P_1} > \mathbf{P}_e$
$\mathbf{P_1} > \mathbf{P}_e$

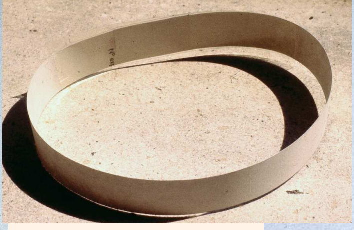


$P_0 < P_e$
$P_1 > P_e$
$P_1 > P_e$
$P_1 > P_e$
P
P_e
p
$e \neq p$
δ
0
δ_e


A, L


A,L
Pu





Ensayo de cinta de papel

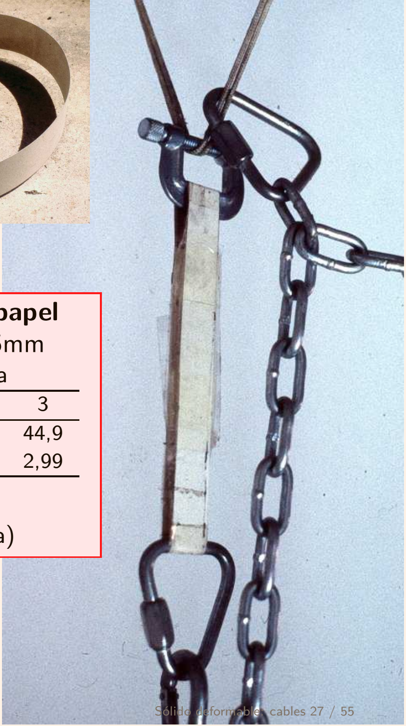
ancho de la cinta: 15mm

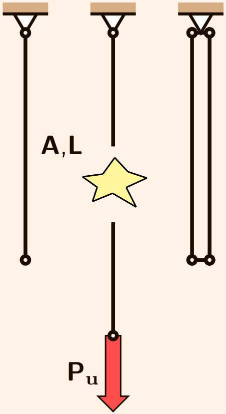
Valores en la rotura

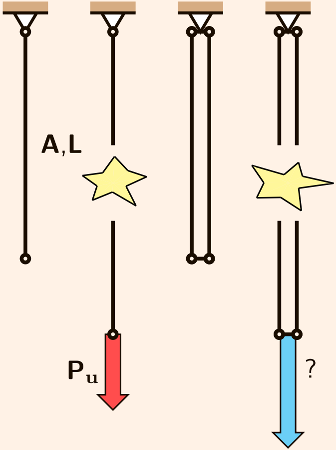
ensayo	1	2	3
P/2 (N)	55,7	56,3	44,9
f (N/mm)	3,71	3,75	2,99

$$\mathbf{f_u \approx 2,99}\,\mathrm{N/mm}$$

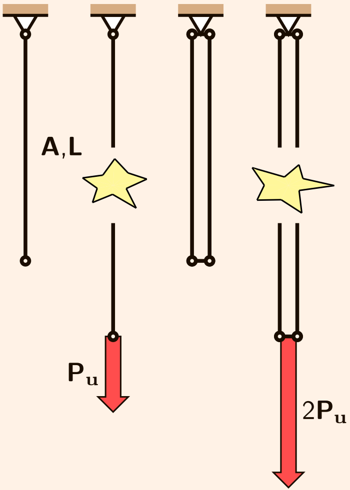
(100 % de confianza)

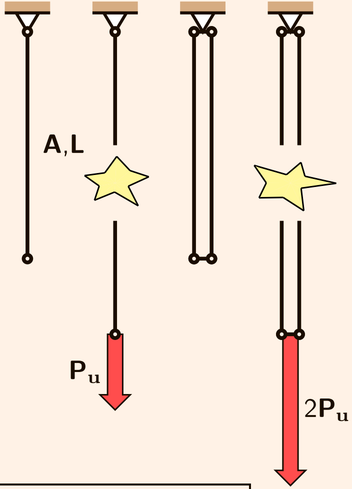



A,L
Pu

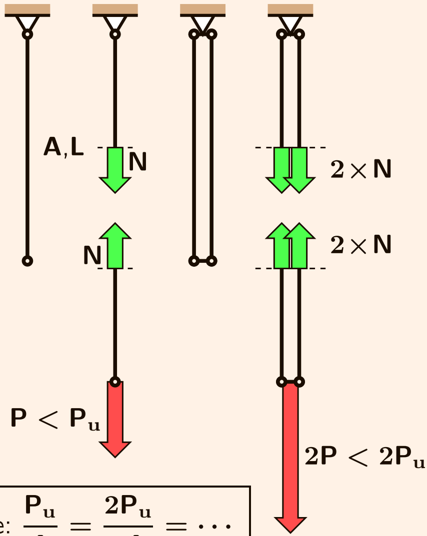


A,L
P$_u$
?


A,L
Pu
2Pu



$$\text{cte: } \frac{\mathbf{P_u}}{\mathbf{A}} = \frac{\mathbf{2P_u}}{\mathbf{2A}} = \cdots$$

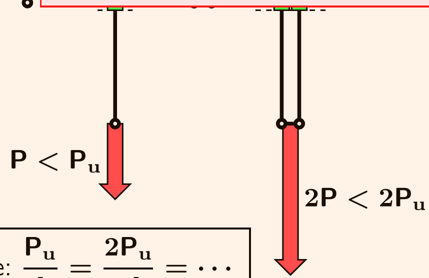


$$\text{cte: } \frac{\mathbf{P_u}}{\mathbf{A}} = \frac{\mathbf{2P_u}}{\mathbf{2A}} = \cdots$$

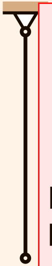
Tensión

$$\bar{\sigma} = \frac{\mathbf{N}}{\mathbf{A}} \qquad \left(\text{en este caso } \frac{\mathbf{P}}{\mathbf{A}}\right)$$

Fuerza por unidad de área de la sección de la barra. N/m^2(un ‘pascal’), N/mm^2, kN/mm^2, etc.



$$\text{cte: } \frac{\mathbf{P_u}}{\mathbf{A}} = \frac{\mathbf{2P_u}}{\mathbf{2A}} = \cdots$$



Tensión

$$\bar{\sigma} = \frac{\mathbf{N}}{\mathbf{A}} \qquad \left(\text{en este caso } \frac{\mathbf{P}}{\mathbf{A}}\right)$$

Fuerza por unidad de área de la sección de la barra. N/m^2(un 'pascal'), N/mm^2, kN/mm^2, etc.

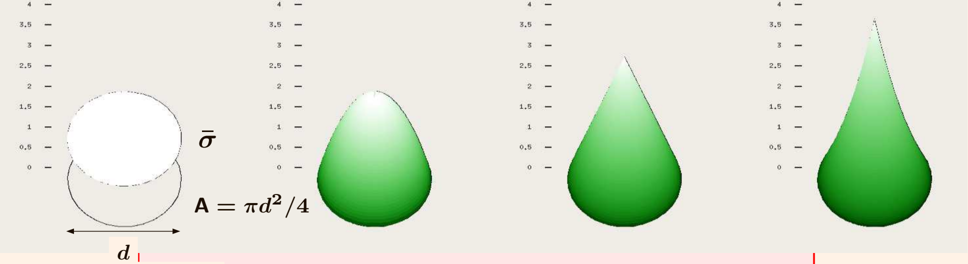
Equilibrio:

$$\mathbf{N} = \int \sigma(x,y)\, d\mathbf{A}$$

$$\bar{\sigma} = \frac{\int \sigma(x,y)\, d\mathbf{A}}{\int d\mathbf{A}} = \frac{\mathbf{N}}{\mathbf{A}}$$

P ·

cte: $\frac{\mathbf{P}_u}{\mathbf{A}} = \frac{2\mathbf{P}_u}{2\mathbf{A}} = \cdots$



$\sigma_{max} =$
$\sigma_{min} = \bar{\sigma}$

$\sigma_{max} = 2\bar{\sigma}$

$\sigma_{max} = 3\bar{\sigma}$

$\sigma_{max} \approx 3{,}94\bar{\sigma}$

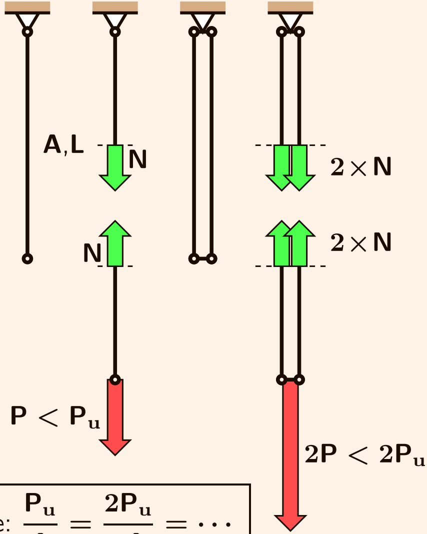
ibrio:

$$N = \int \sigma(x,y)\,dA$$

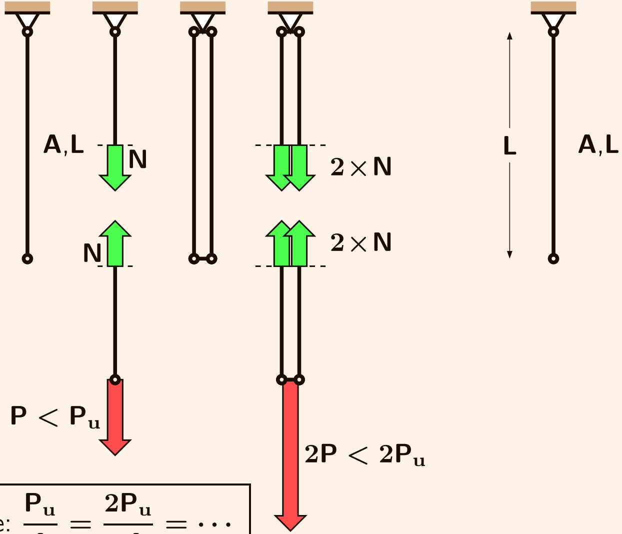
$$\bar{\sigma} = \frac{\int \sigma(x,y)\,dA}{\int dA} = \frac{N}{A}$$

P

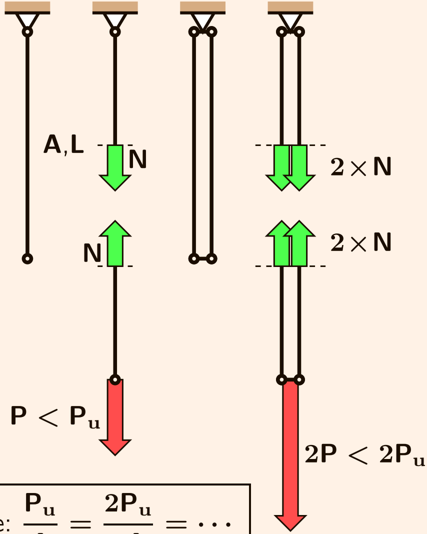
cte: $\frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$



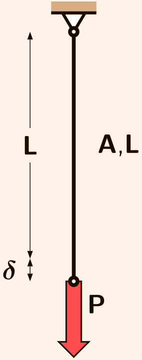
$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$

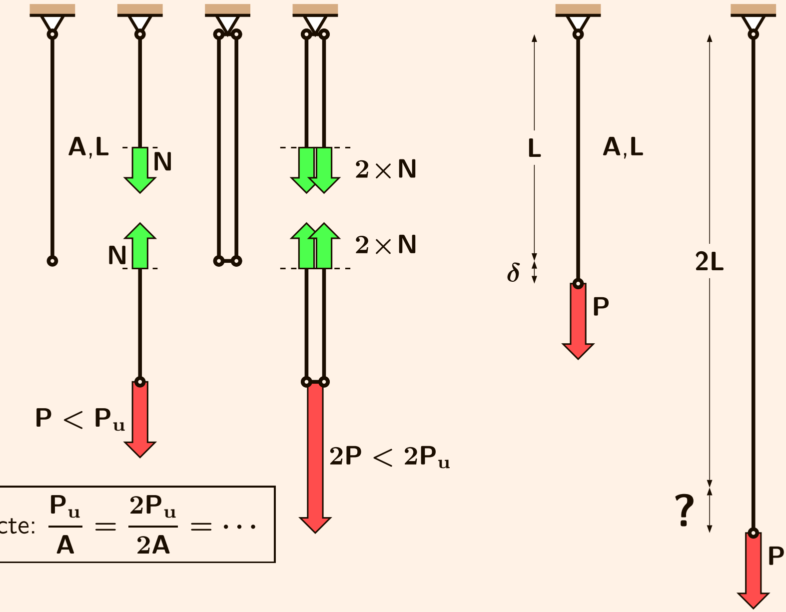


$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$

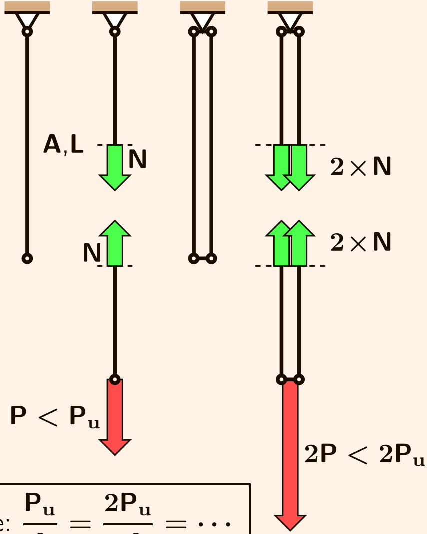


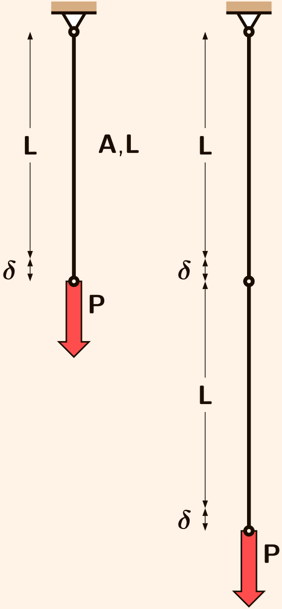
$$\text{cte: } \frac{\mathbf{P_u}}{\mathbf{A}} = \frac{\mathbf{2P_u}}{\mathbf{2A}} = \cdots$$



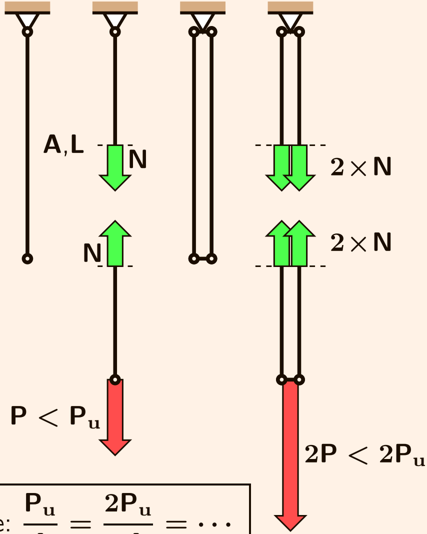


$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$

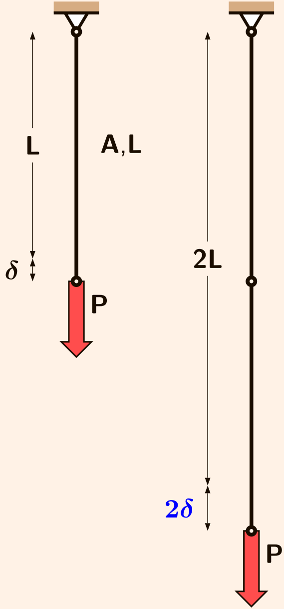


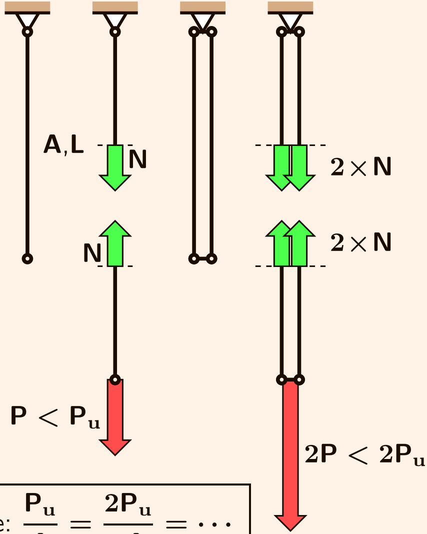


$$\text{cte: } \frac{\mathbf{P_u}}{\mathbf{A}} = \frac{\mathbf{2P_u}}{\mathbf{2A}} = \cdots$$

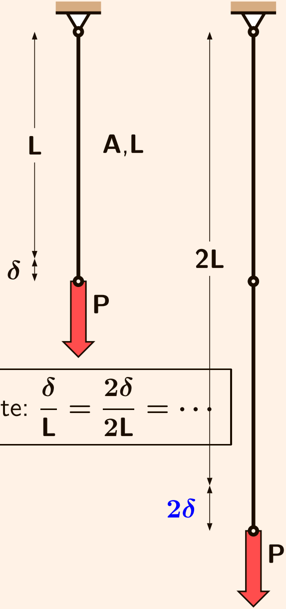


$$\text{cte: } \frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$$





cte: $\frac{P_u}{A} = \frac{2P_u}{2A} = \cdots$

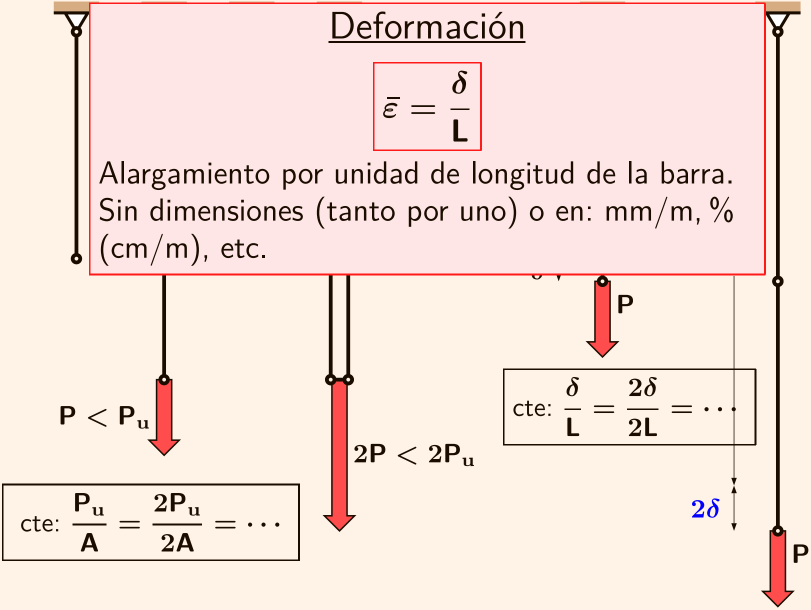


cte: $\frac{\delta}{L} = \frac{2\delta}{2L} = \cdots$

Deformación

$$\bar{\varepsilon} = \frac{\delta}{L}$$

Alargamiento por unidad de longitud de la barra. Sin dimensiones (tanto por uno) o en: mm/m, % (cm/m), etc.



Deformación

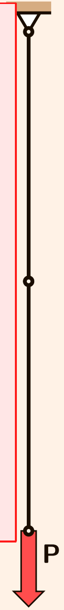
$$\bar{\varepsilon} = \frac{\delta}{\mathsf{L}}$$

Alargamiento por unidad de longitud de la barra. Sin dimensiones (tanto por uno) o en: mm/m, % (cm/m), etc.

Compatibilidad:

$$\delta = \int \varepsilon(z)\,\mathrm{d}z$$

$$\bar{\varepsilon} = \frac{\int_{[]L} \varepsilon(z)\,\mathrm{d}z}{\int_{[]L}\,\mathrm{d}z} = \frac{\delta}{\mathsf{L}}$$

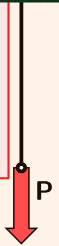


$$\delta = \int \varepsilon(z)\,\mathrm{d}z$$

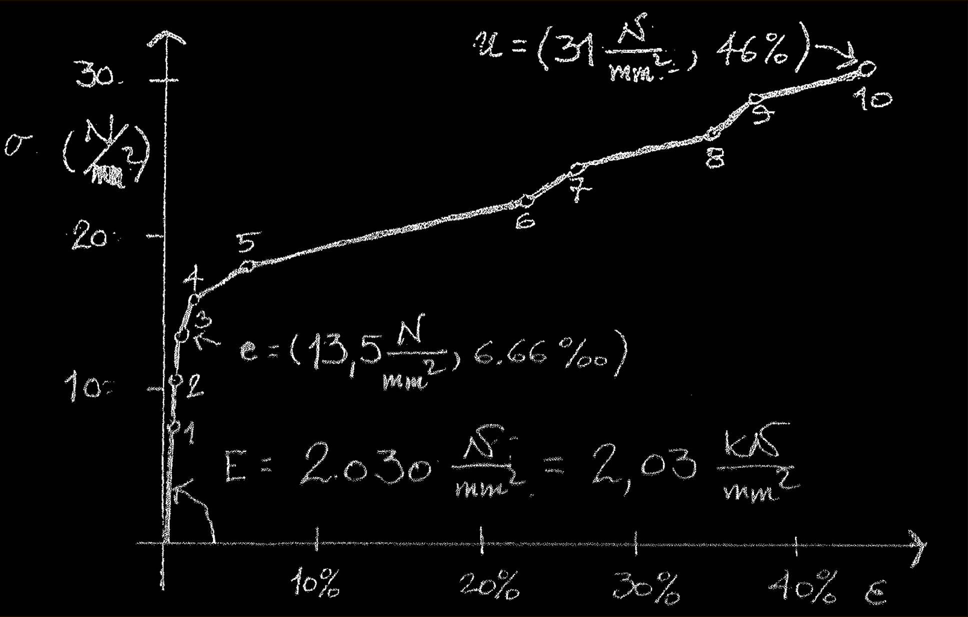
P

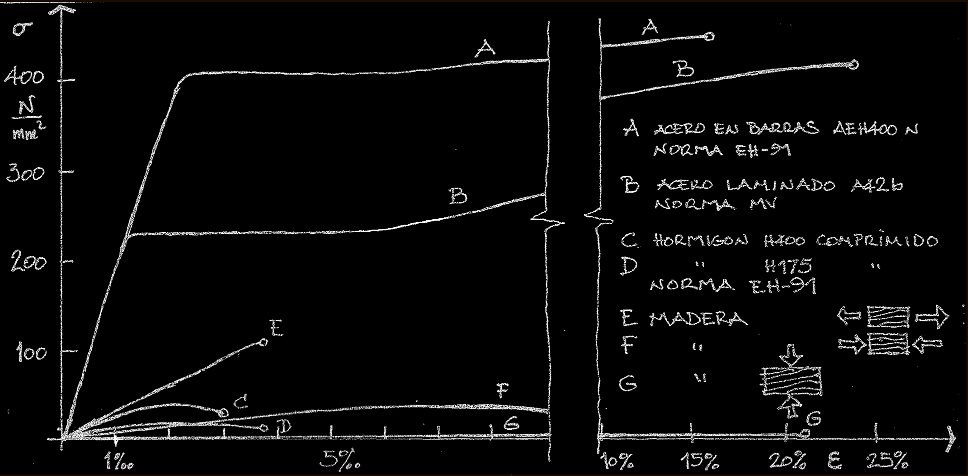
$$\bar{\varepsilon} = \frac{\int_{[]L} \varepsilon(z)\,\mathrm{d}z}{\int_{[]L} \mathrm{d}z} = \frac{\delta}{\mathsf{L}}$$

cte: $\frac{\mathsf{P}}{\mathsf{A}}$ … $\mathsf{2A}$



video: http://www.aq.upm.es/Departamentos/Estructuras/e96-290/doc/


u = (31 N/mm², 46%)
σ (N/mm²)
30
20
10
1
2
3
4
5
6
7
8
9
10
e = (13,5 N/mm², 6.66 ‰)
E = 2.030 N/mm² = 2,03 kN/mm²
10%
20%
30%
40%
ε


σ
400
N/mm²
300
200
100
1‰
5‰
10%
15%
20%
ε
25%
A
B
C
D
E
F
G
A ACERO EN BARRAS AEH400 N
NORMA EH-91
B ACERO LAMINADO A42b
NORMA MV
C HORMIGON H400 COMPRIMIDO
D " H175 "
NORMA EH-91
E MADERA
F "
G "

Marco Polo describe un puente piedra a piedra.

¿Pero cual es la piedra que sostiene el puente? —pregunta Kublai Kan.

El puente no está sostenido por esta o aquella piedra, —responde Marco— sino por la línea del arco que forman.

Kublai Kan queda silencioso, reflexionando. De repente, dice:— ¿Por qué me hablas entonces de las piedras? Es sólo el arco lo que me importa.

Polo responde:— Sin piedras no habría arco.

ITALO CALVINO

- Comportamiento del material
  - Ley de Hooke: $\mathbf{P} = \mathbf{K}\boldsymbol{\delta}$ si $\mathbf{P} < \mathbf{P}_p$; en otro caso:
  - Plasticidad 'perfecta': $\mathbf{P} = \mathbf{P_u}$ si $\boldsymbol{\delta}_e < \boldsymbol{\delta} < \boldsymbol{\delta}_\mathbf{u}$
- Mientras no se produzca la rotura:
  - Equilibrio: $\mathbf{N} = \mathbf{P}$ (en el caso de cables verticales) y $\boldsymbol{\sigma} = \dfrac{\mathbf{N}}{\mathbf{A}}$ (es decir que $\mathbf{N} = \boldsymbol{\sigma}\mathbf{A}$)
  - Compatibilidad: $\boldsymbol{\delta} = \varepsilon\mathbf{L}$ (y también $\varepsilon = \dfrac{\boldsymbol{\delta}}{\mathbf{L}}$)

En el periodo proporcional:

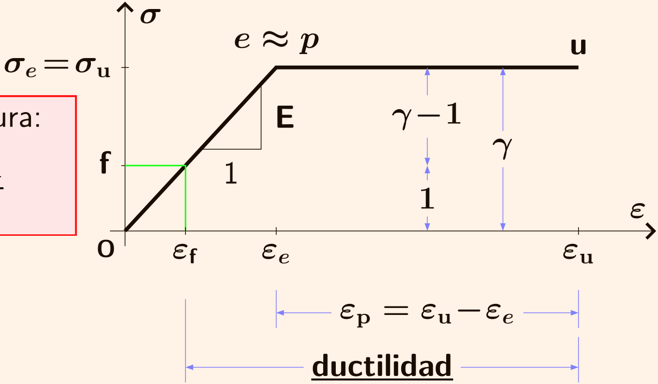
$$\mathbf{K} = \frac{\mathbf{N}}{\boldsymbol{\delta}} = \frac{\boldsymbol{\sigma}\mathbf{A}}{\varepsilon\mathbf{L}} = \frac{\mathbf{EA}}{\mathbf{L}} \quad \Rightarrow \quad \mathbf{E} = \frac{\boldsymbol{\sigma}}{\varepsilon}$$

Modelo elasto-plástico perfecto de los materiales

$$\boldsymbol{\sigma}(\varepsilon)=\begin{cases} \mathbf{E}\varepsilon & \text{si } \varepsilon \leq \varepsilon_e \\ \boldsymbol{\sigma}_{\mathbf{u}} & \text{si } \varepsilon_e < \varepsilon \leq \varepsilon_{\mathbf{u}} \\ 0 & \text{si } \varepsilon_{\mathbf{u}} < \varepsilon \end{cases}$$

Tensión segura:

$$\mathbf{f} = \frac{\boldsymbol{\sigma}_{\mathbf{u}}}{\gamma}$$



Características importantes de los materiales para las estructuras de los edificios:

- **Ductilidad:** cuanto mayor deformación antes de la rotura, **¡mejor!**

  (cuanto menor, mayor margen de seguridad γ habrá que adoptar)

- **Fiabilidad**

  (cuanto menor, mayor margen de seguridad γ habrá que adoptar)

- **Costes físicos específicos**: energía fósil incorporada (*embodied energy*), emisiones contaminantes, etc, por unidad de cada propiedad mecánica de interés (rigidez $\mathbf{E}$, tensión segura, $\mathbf{f}$, etc). **¡Cuánto menor, mejor!**

$$\left\{\frac{\mathcal{C}}{\mathbf{E}}, \frac{\mathcal{C}}{\mathbf{f}}, \ldots\right\}$$

Modelo ‘cable’

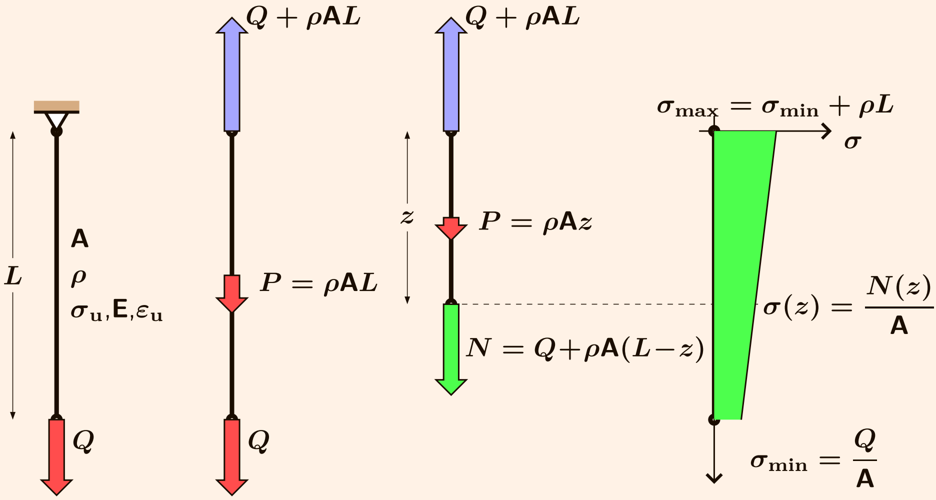
En el estado proporcional, sin superar el límite ‘elástico’:

$$\varepsilon = \frac{\delta}{\mathsf{L}}; \qquad \sigma = \mathsf{E}\varepsilon; \qquad \mathsf{N} = \sigma \mathsf{A}; \qquad \text{si } \varepsilon \leq \varepsilon_e.$$

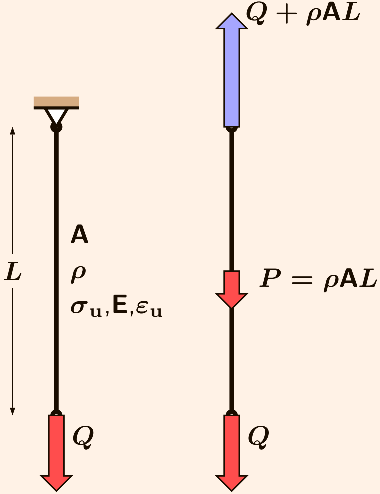
$$\mathsf{K}_{\text{cable}} = \frac{\mathsf{N}}{\delta} = \frac{\sigma \mathsf{A}}{\varepsilon \mathsf{L}} = \frac{\mathsf{EA}}{\mathsf{L}}$$

En general:

$$\mathsf{N}(\delta) = \begin{cases} 0 & \text{si } \varepsilon < 0 & \text{acortamiento} \\ \mathsf{K}\delta & \text{si } 0 \leq \varepsilon \leq \varepsilon_e & \text{e. proporcional} \\ \sigma_{\mathbf{u}}\mathsf{A} & \text{si } \varepsilon_e \leq \varepsilon \leq \varepsilon_{\mathbf{u}} & \text{e. plástico} \\ 0 & \text{si } \varepsilon_{\mathbf{u}} < \varepsilon & \text{rotura} \end{cases}$$

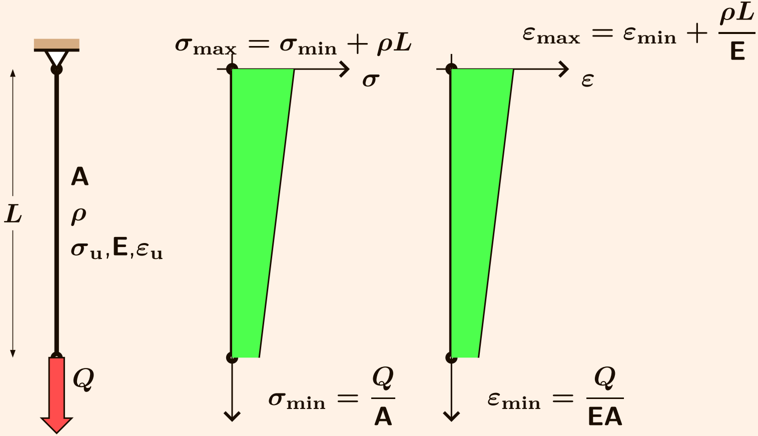


$Q+\rho\mathsf{A}L$
$Q+\rho\mathsf{A}L$
$\sigma_{\mathrm{max}}=\sigma_{\mathrm{min}}+\rho L$
σ
L
A
ρ
σ_{u},E,ε_{u}
$P=\rho\mathsf{A}L$
z
$P=\rho\mathsf{A}z$
$\sigma(z)=\frac{N(z)}{\mathsf{A}}$
$N=Q+\rho\mathsf{A}(L-z)$
Q
Q
$\sigma_{\mathrm{min}}=\frac{Q}{\mathsf{A}}$



Resistencia

Rigidez


L
A
ρ
σ_u,E,ε_u
Q
$\sigma_{max} = \sigma_{min} + \rho L$
σ
$\sigma_{min} = \frac{Q}{A}$
$\varepsilon_{max} = \varepsilon_{min} + \frac{\rho L}{E}$
ε
$\varepsilon_{min} = \frac{Q}{EA}$

Sólido deformable: cables

Mariano Vázquez Espí

GIAU+S (UPM)
Grupo de Investigación en Arquitectura, Urbanismo y Sostenibilidad
Universidad Politécnica de Madrid
http://habitat.aq.upm.es/gi

Edición del 1 de octubre de 2010
Compuesto con *free software*:
GNULinux/LaTeX/dvips/ps2pdf

Si no se lo creen. . .