

# Diagramas de esfuerzos

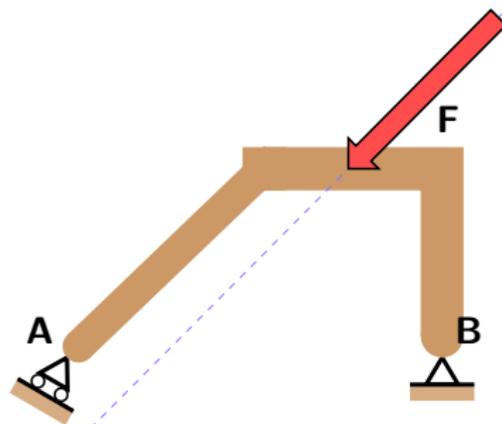
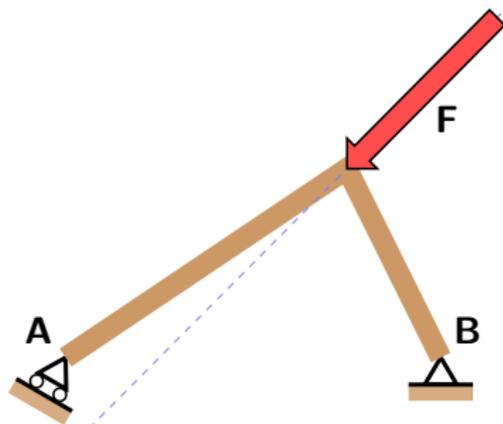
(Funiculares como diagramas)

**Mariano Vázquez Espí**

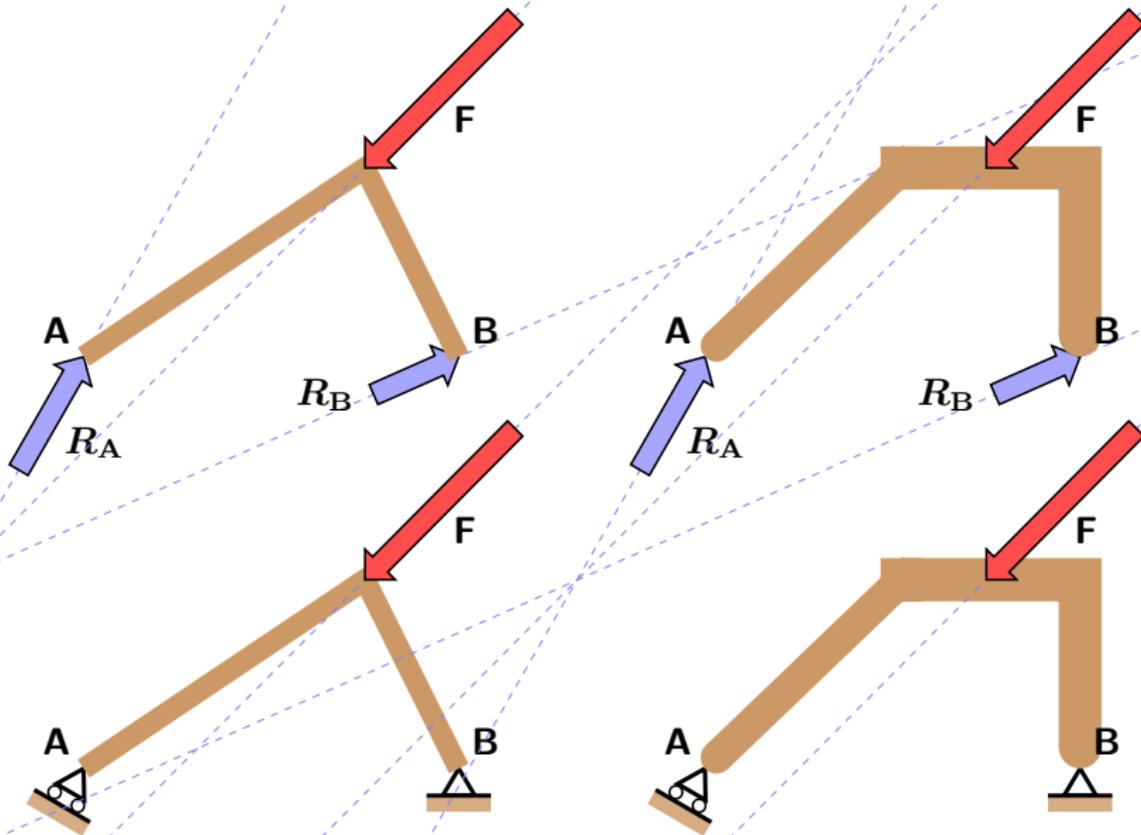
**Ondara, 22 de octubre de 2010.**

# Fuerzas internas en cortes imaginarios

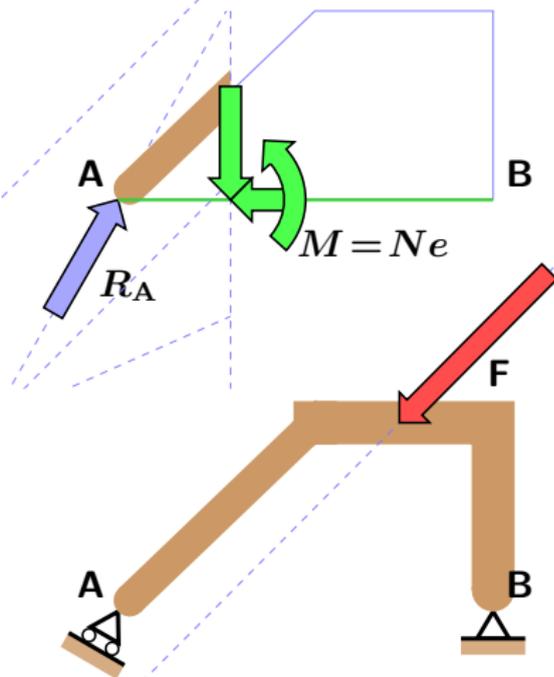
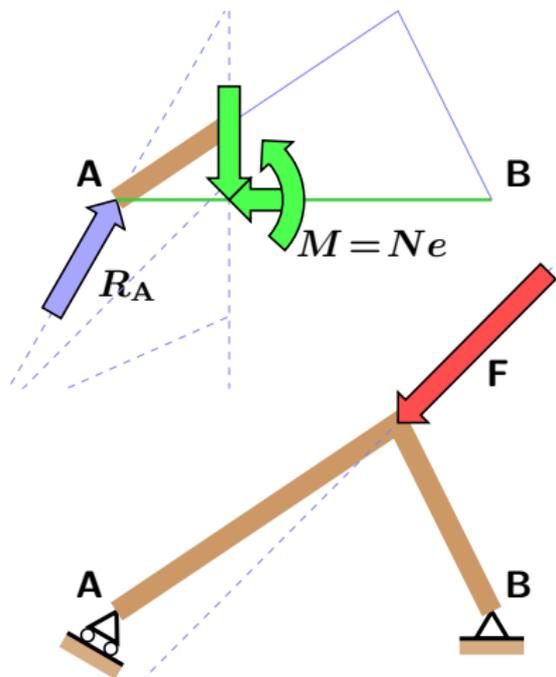
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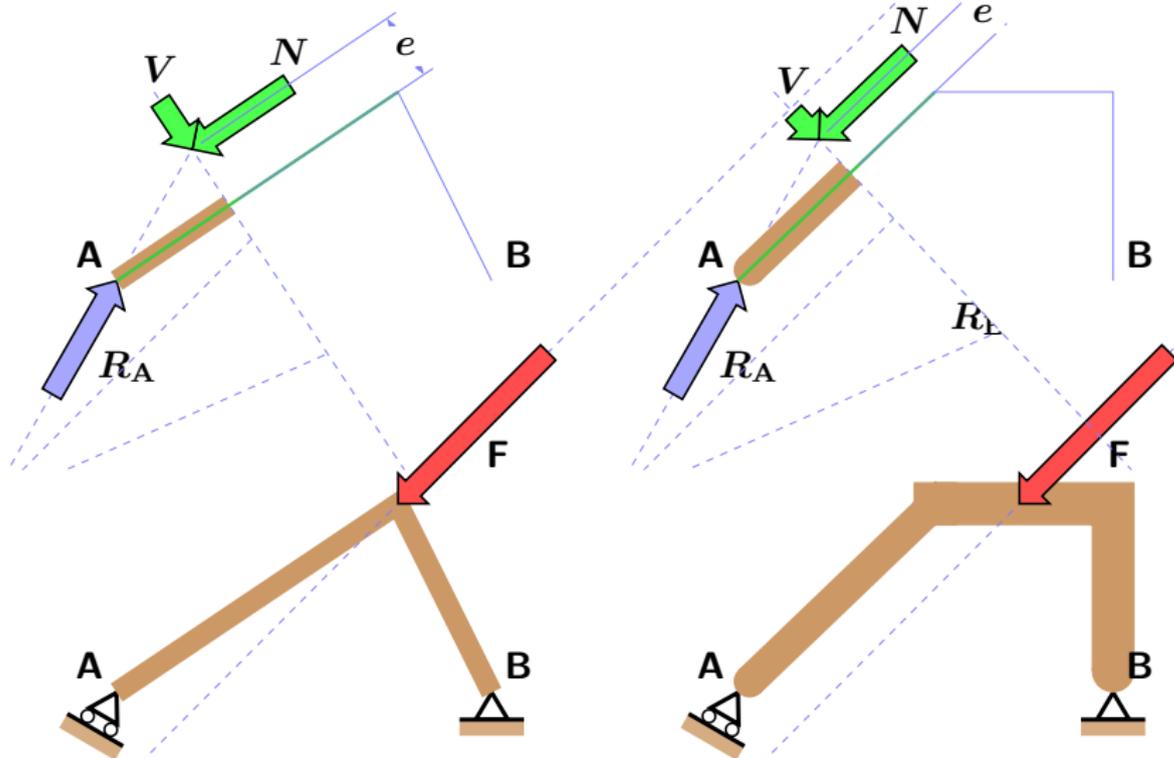
# Fuerzas internas en cortes imaginarios



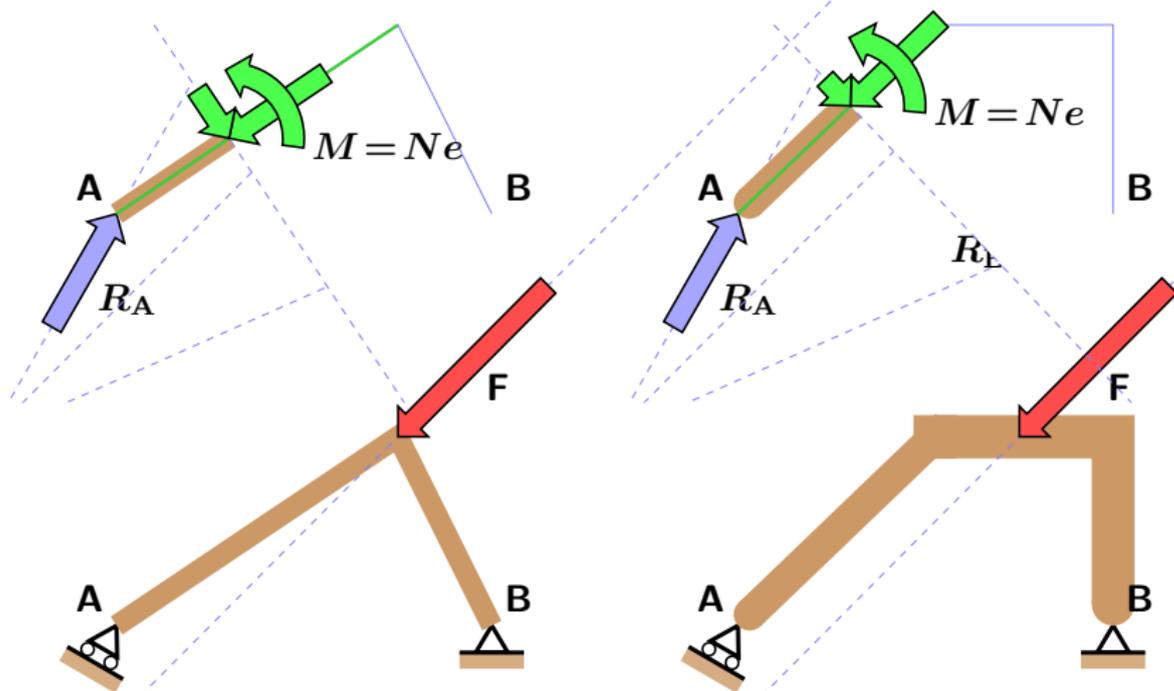
# Fuerzas internas en cortes imaginarios



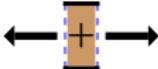
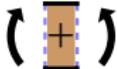
# Fuerzas internas en cortes imaginarios



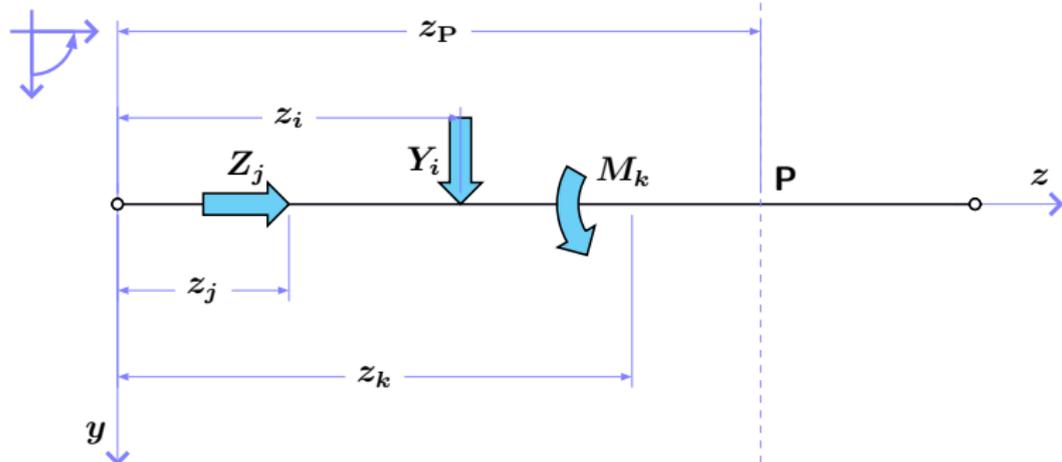
# Fuerzas internas en cortes imaginarios



# Solicitaciones y esfuerzos

Solicitud	Esfuerzos		
	Longitudinal Normal Axil	Transversal Cortante	Par Flector Momento flector
Tracción simple	$N$	—	—
Flexión simple	—	$V$	$M$
Flexión compuesta	$N$	$V$	$M$
Tracción compuesta			
Compresión compuesta	$N$	—	$M$
Compresión simple			
Cizalladura	—	$V$	—
Flexión pura	—	—	$M$
	$\sigma$ constante 	$\tau$ 	$\sigma$ variable 

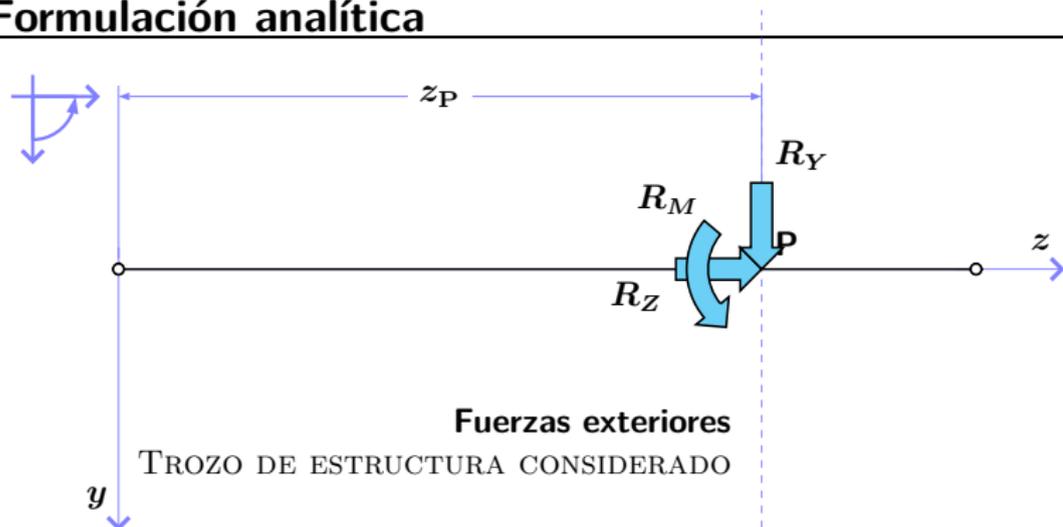
## Formulación analítica



$$R_Z(z_P) = \sum_{z_j \leq z_P} Z_j \quad R_Y(z_P) = \sum_{z_i \leq z_P} Y_i$$

$$R_M(z_P) = \sum_{z_i \leq z_P} Y_i \cdot (z_P - z_i) + \sum_{z_k \leq z_P} M_k$$

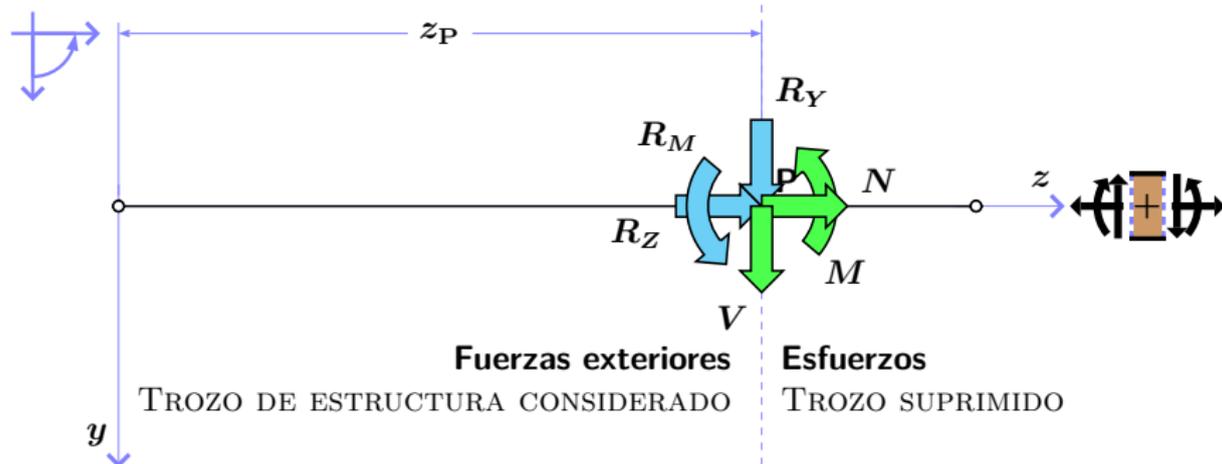
## Formulación analítica



$$R_Z(z_P) = \sum_{z_j \leq z_P} Z_j \quad R_Y(z_P) = \sum_{z_i \leq z_P} Y_i$$

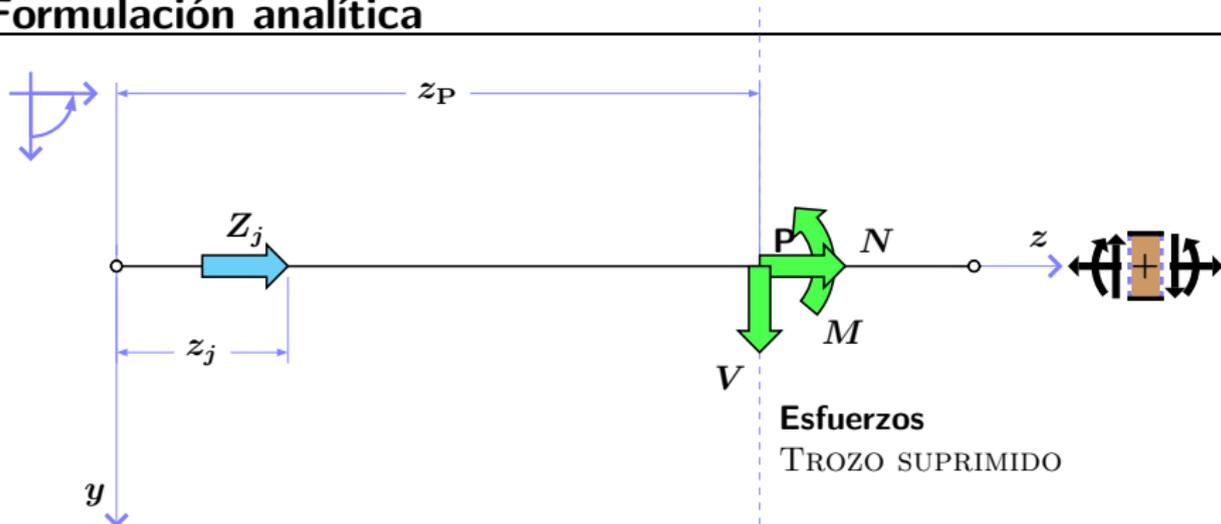
$$R_M(z_P) = \sum_{z_i \leq z_P} Y_i \cdot (z_P - z_i) + \sum_{z_k \leq z_P} M_k$$

## Formulación analítica



$$\begin{cases} R_Z(z_P) + N(z_P) = 0 \\ R_Y(z_P) + V(z_P) = 0 \\ R_M(z_P) + M(z_P) = 0 \end{cases} \Rightarrow \begin{cases} N(z_P) = -R_Z(z_P) \\ V(z_P) = -R_Y(z_P) \\ M(z_P) = -R_M(z_P) \end{cases}$$

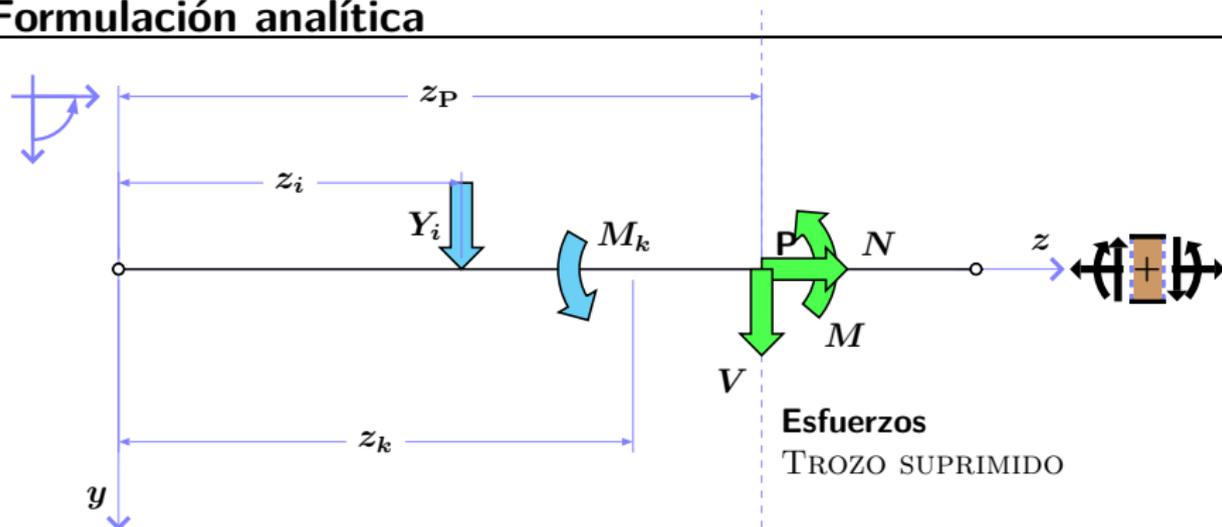
## Formulación analítica



Ecuación para el diagrama de esfuerzos normales:

$$N(z_P) = - \sum_{z_j \leq z_P} Z_j$$

## Formulación analítica

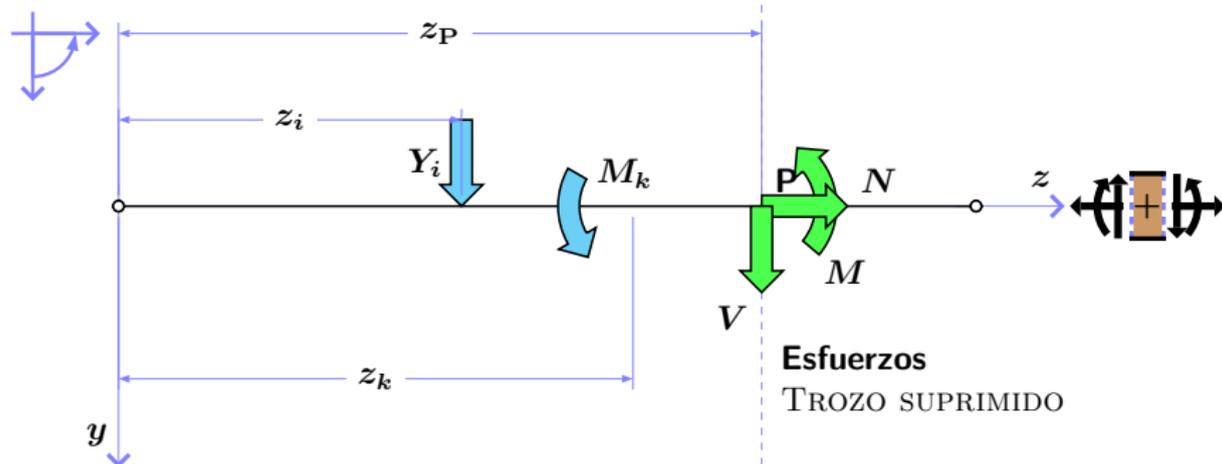


Ecuaciones para los diagramas de la flexión simple:

$$V(z_P) = - \sum_{z_i \leq z_P} Y_i$$

$$M(z_P) = - \sum_{z_i \leq z_P} Y_i \cdot (z_P - z_i) - \sum_{z_k \leq z_P} M_k$$

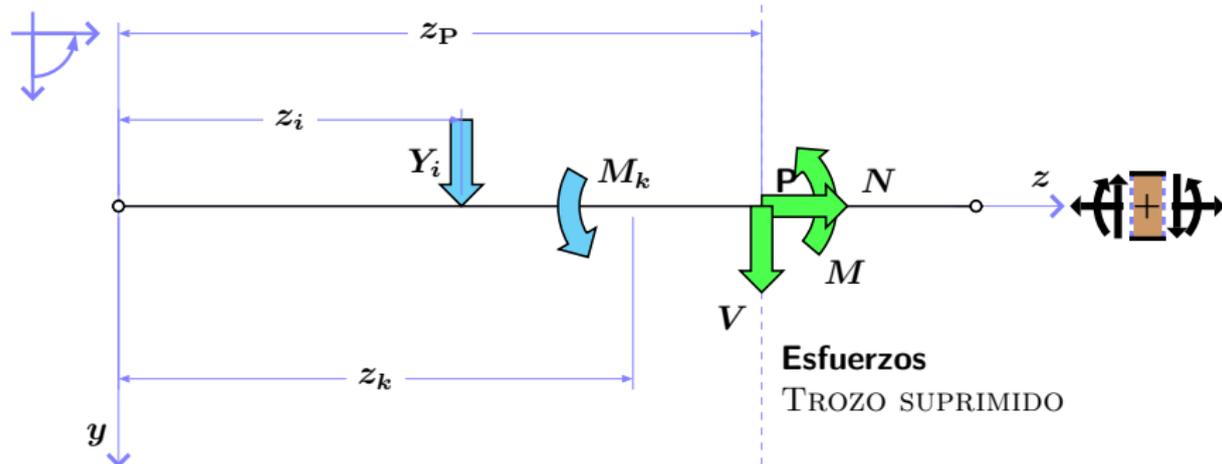
## Formulación analítica



$$V(z_P) = - \sum_{z_i \leq z_P} Y_i$$

$$\frac{\partial M(z_P)}{\partial z_P} = - \sum_{z_i \leq z_P} Y_i = V(z_P)$$

## Formulación analítica

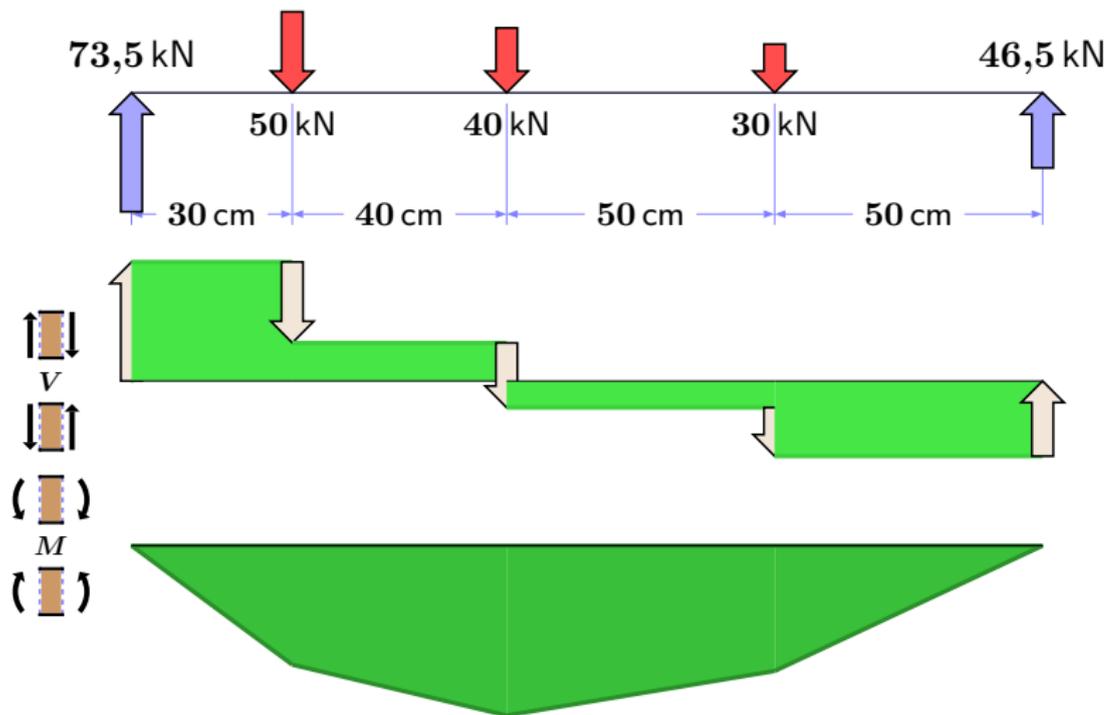


$$M(z) = H \cdot y(z) \quad \frac{\partial M(z)}{\partial z} = V(z) = H \cdot y'(z)$$

$$M(z_c) = - \int_0^{z_c} p_y(z_c - z) dz \quad V(z_c) = - \int_0^{z_c} p_y dz$$

$$\frac{\partial^2 M(z_c)}{\partial z_c^2} = \frac{\partial V(z_c)}{\partial z_c} = -p_y$$

## Tres cargas entre dos apoyos



# Diagramas de esfuerzos (Funiculares como diagramas)

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Compuesto con *free software*:  
GNU/Linux/L<sup>A</sup>T<sub>E</sub>X/dvips/ps2pdf

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