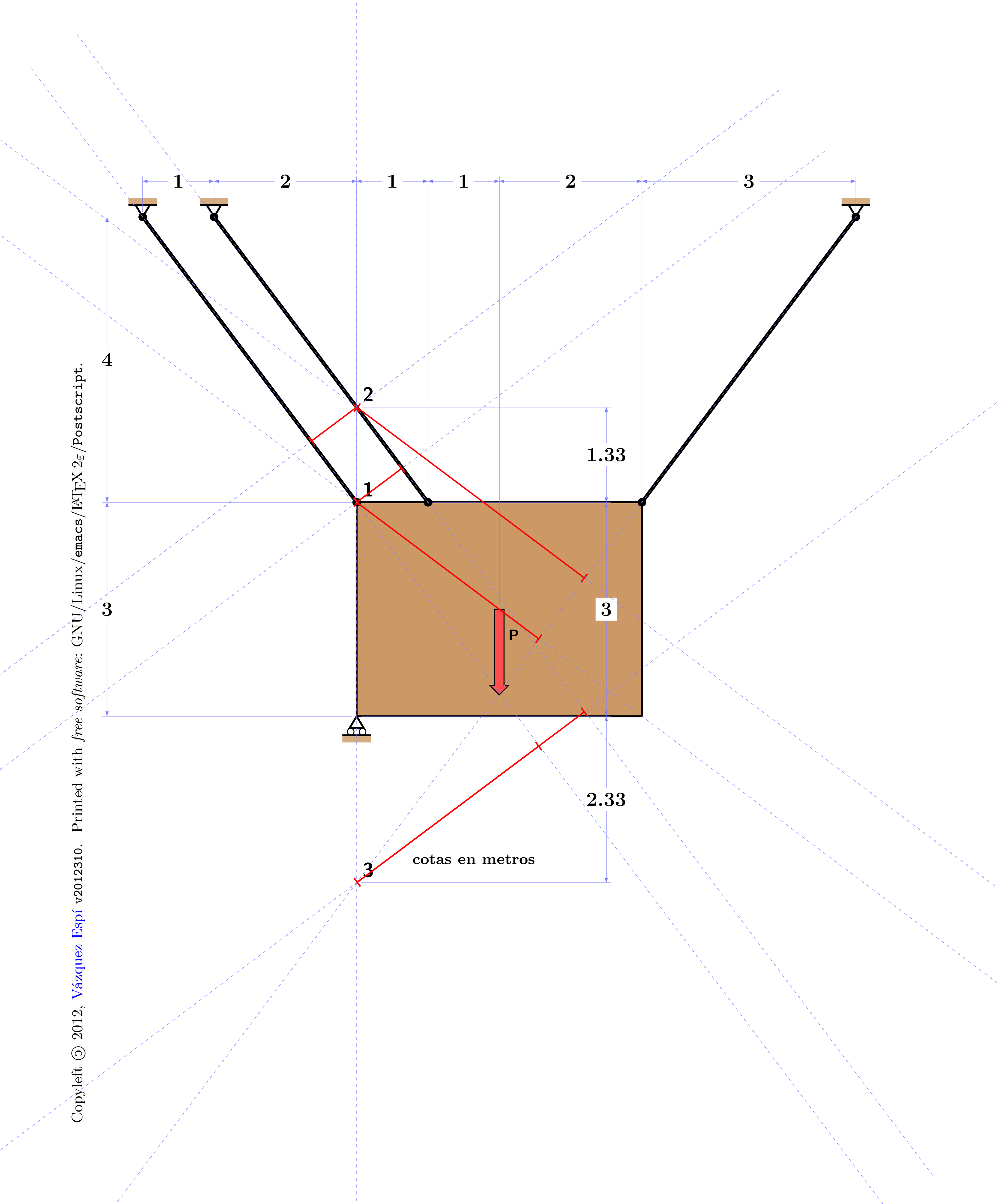
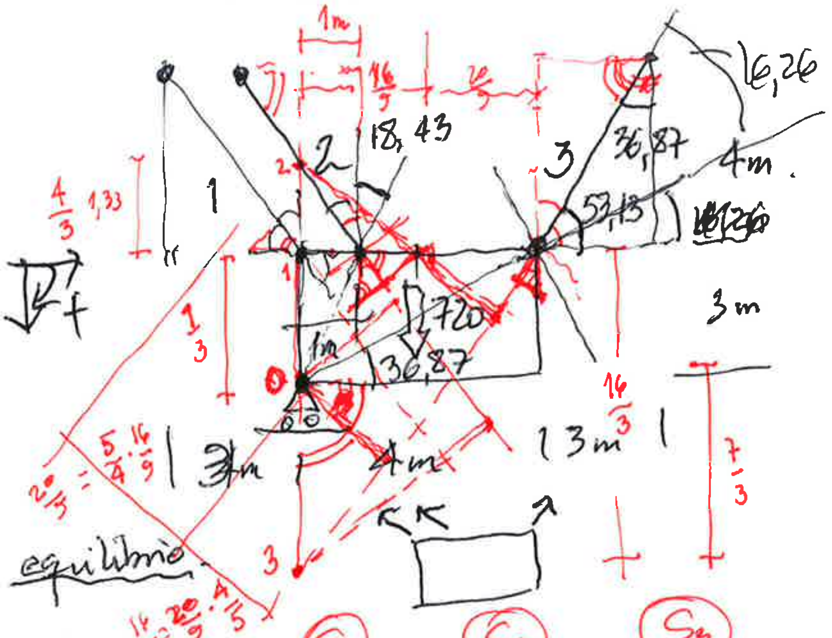
Estructura con dos grados de libertad y tres modos de deformación



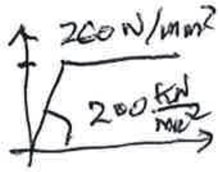


barras 2∅40 mm

$A = 2\times 1257\,mm^2 = 2513\,mm^2$

$$k = \frac{2513\,mm^2 \times 200\,kN/mm^2}{5000\,mm} = 100\,\frac{kN}{mm}$$

$N_u = 653\,kN$



$\varepsilon_u = 10\,mm/m$
$\delta_u = 50\,mm$

$$0 \quad -N_1\frac{3}{5} - N_2\frac{3}{5} + N_3\frac{3}{5} = 0$$

$$1440\,mkN - N_1\cdot\frac{3}{5}\times 3m - N_2\left(\frac{3}{5}\times 3m + \frac{4}{5}\times 4m\right) + N_3\left(\frac{3}{5}\times 3m - \frac{4}{5}\times 4m\right) = 0$$

$$\begin{bmatrix} 0 \\ 1440 \end{bmatrix} = \underbrace{\begin{bmatrix} 3/5 & 3/5 & -3/5 \\ 9m/5 & 13m/5 & +7m/5 \end{bmatrix}}_{B^T} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$$

compatibilidad

	u	θ
δ_1	3/5	3m × 3/5
δ_2	3/5	3,16m × 0,82
δ_3	−3/5	5m × 0,28

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 3/5 & 9m/5 \\ 3/5 & 13m/5 \\ -3/5 & 7m/5 \end{bmatrix}}_{B} \begin{bmatrix} u \\ \theta \end{bmatrix}$$

rigidez de elementos

$k_i = 101\,\frac{kN}{mm}$

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times 101\,\frac{kN}{mm} \times \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

SOLIDO DEFORMABLE (II). EJEMPLO COMPLICADO 26/2/2003 ②

SOLUCIÓN PERIODO PROPORCIONAL.

$$a = \begin{bmatrix} -2{,}65\,mm \\ 1{,}59\,mm/m \end{bmatrix} \quad S = \begin{bmatrix} 129\,kN \\ 257\,kN \\ 386\,kN \end{bmatrix} \quad \sigma = \begin{bmatrix} 51{,}2 \\ 102 \\ 153 \end{bmatrix} \frac{N}{mm^2}$$

LÍMITE ELÁSTICO.

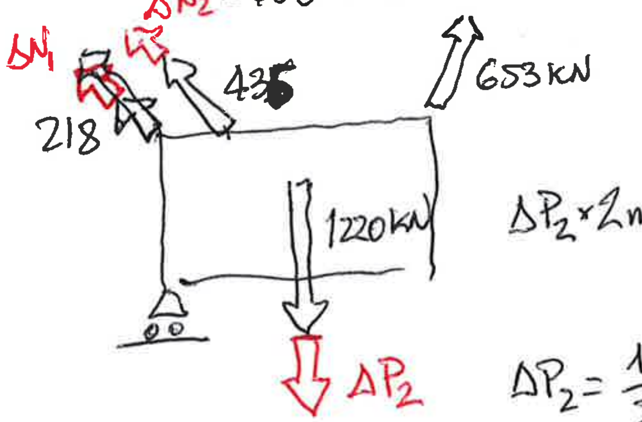
$$\min_i \left(\frac{\sigma_e}{\sigma}\right)_i = \frac{\cancel{153}\ 260}{153} = 1{,}69 \quad \boxed{P_e = 1.220\,kN} = P_1$$

$$= \frac{\cancel{358}}{386} = \frac{653}{386}$$

$$a = \begin{bmatrix} -4{,}49\,mm \\ 2{,}69\,mm/m \end{bmatrix} \quad S = \begin{bmatrix} 218 \\ 436 \\ 653 \end{bmatrix} kN$$

CARGA MAYOR QUE EL LÍMITE ELÁSTICO.

$\Delta N_2 = 653 - 435\ kN = 217\,kN$



análisis isostático.

$\Delta N_2 = 217\,kN$

$\Delta N_1 = -\Delta N_2 \quad (\Sigma F_H = 0)$

$$\Delta P_2 \times 2m = \Delta N_1 \times \frac{9m}{5} + \Delta N_2 \times \frac{13m}{5} \quad (\Sigma M = 0)$$

$$\Delta P_2 = \frac{1}{2}\Delta N_2\left(\frac{4}{5}\right) = \frac{2}{5}\,217\,kN = 86{,}8\,kN.$$

$\delta_1 = 0{,}01\,mm$

$\delta_2 = 6{,}47\,mm.$

$\Delta N_1 = -217\,kN.$

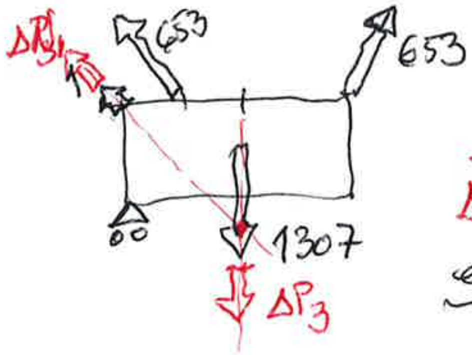
$$\boxed{\begin{array}{ll} N_1 \simeq 1\,kN & \\ N_2 = 653\,kN & P_2 = 1307\,kN \\ N_3 = 653\,kN & \end{array}}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 0{,}60 & 1{,}8\,m \\ 0{,}60 & 2{,}6\,m \end{bmatrix} \begin{bmatrix} a \\ \theta \end{bmatrix}$$

$$\begin{bmatrix} a \\ \theta \end{bmatrix} = \begin{bmatrix} -24{,}21\,mm \\ 8{,}08\,mm/m \end{bmatrix}$$

$\delta_3 = 25{,}83\,mm. < \delta_M = 50\,mm.$

¿CARGA MAYOR QUE P_2?



$\vec{\Delta P_3} + \vec{\Delta N_1} \neq 0$

equilibrio imposible.

ROTURA FÍSICA.

$\delta_1 = 0{,}01$ mm.

$\delta_3 = 50$ mm.

$$\begin{bmatrix} \delta_1 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 0{,}60 & 1{,}80\,m \\ -0{,}60 & 1{,}40\,m \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix}$$

$$\begin{bmatrix} u \\ \theta \end{bmatrix} = \begin{bmatrix} -46{,}9\ mm \\ 15{,}6\ mm/m \end{bmatrix}$$

(arc tan $0{,}0156 \simeq 0{,}016$
sin $0{,}016 \sim 0{,}016$)

$\delta_2 = 12{,}5$ mm.

ANÁLISIS DIRECTO DE LA ROTURA. $g=2$ $s=3$ $g-1=1$

$\boxed{d\delta_3 = 0}$ $\quad -\frac{3}{5}u + 1{,}4\,m\,d\theta = 0 \qquad u = 2{,}33\,\theta.$

$$\begin{bmatrix} du \\ d\theta \end{bmatrix} = \begin{bmatrix} 2{,}33\,m \\ 1 \end{bmatrix} d\theta.$$

$$\begin{bmatrix} d\delta_1 \\ d\delta_2 \\ d\delta_3 \end{bmatrix} = \begin{bmatrix} 3{,}2\,m \\ 4\,m \\ 0 \end{bmatrix} d\theta.$$

$W_{ext} = P_u \times 2m \times d\theta$

$W_{int} = 653\,kN\,(3{,}2m + 4m)\,d\theta$

$W_{ext} = W_{int}$

$P_u = \frac{1}{2m}\, 653\,kN\,(3{,}2m + 4m) = 2351.$

$\boxed{d\delta_2 = 0}$

$0{,}6\,du + 2{,}6\,d\theta = 0 \qquad du = -4{,}33\,m\,d\theta$

$$\begin{bmatrix} du \\ d\theta \end{bmatrix} = \begin{bmatrix} -4{,}33\,m \\ 1 \end{bmatrix} d\theta$$

$dW_{ext} = dW_{int}.$

$$\begin{bmatrix} d\delta_1 \\ d\delta_2 \\ d\delta_3 \end{bmatrix} = \begin{bmatrix} -0{,}8 \\ 0 \\ 4 \end{bmatrix} d\theta$$

$\cancel{P_u \times 2m\,d\theta = 653\,kN\,(-0{,}8m + 4m)\,d\theta}$

$\cancel{P_u = 1044.}$

$P_u \times 2m\,d\theta = \{0\,kN(-0{,}8\,m) + 653\,kN \times 4m\}\,d\theta$

$P_u = 1306\,kN$

$\boxed{d\delta_1 = 0}$

$\cancel{-0{,}6\,du + 1{,}4\,d\theta}$

$0{,}6\,du + 1{,}8\,d\theta = 0 \qquad du = -3\,d\theta$

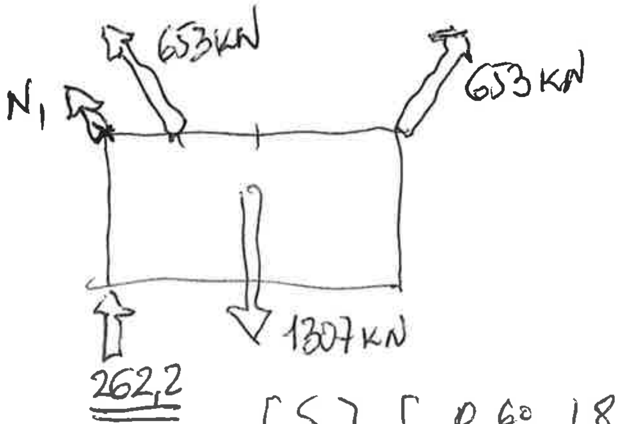
$$\begin{bmatrix} du \\ d\theta \end{bmatrix} = \begin{bmatrix} -3m \\ 1 \end{bmatrix} d\theta$$

$$\begin{bmatrix} d\delta_1 \\ d\delta_2 \\ d\delta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0{,}8m \\ 3{,}2m \end{bmatrix} d\theta$$

$P_u \times 2m\,d\theta = 653\,kN\,(0{,}8m + 3{,}2m)\,d\theta$

$P_u = 2 \times 653\,kN = 1307\,kN.$

ANÁLISIS DE LA SITUACIÓN DE COLAPSO. $\boxed{d\delta_1 = 0}$



$\Sigma F_H = 0 \Rightarrow \boxed{N_1 = 0}$

$\Sigma F_V = 0 \Rightarrow R = 262{,}2$

$\delta_1 = 0$

$(\delta_2 \text{ ó } \delta_3) = 6{,}47\,mm.$

a la vista de $[B] \times [C]$

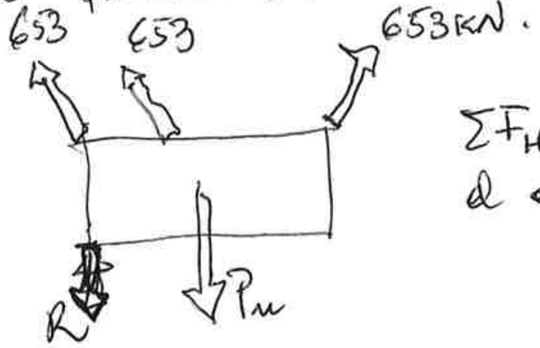
es $\delta_2 = 6{,}47\,mm.$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 0{,}60 & 1{,}8m \\ 0{,}60 & 2{,}6m \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ \theta \end{bmatrix} = \begin{bmatrix} -24\,mm \\ 8{,}09\,mm/m \end{bmatrix}$$

$\delta_3 = 25{,}88\,mm/m.$

MECANISMO DE COLAPSO CON $S_K=0$

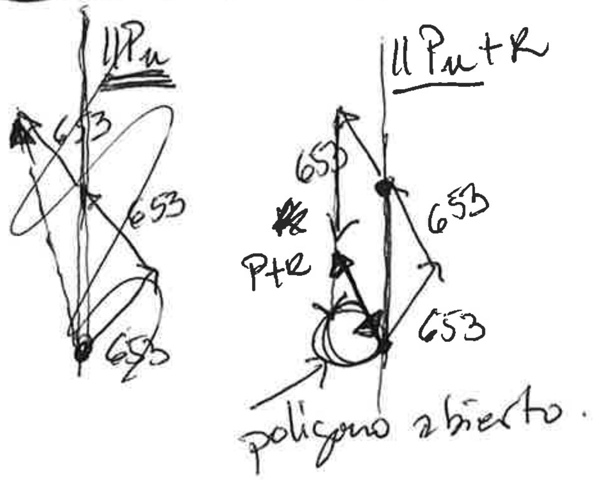
(todos los cables plastifican).



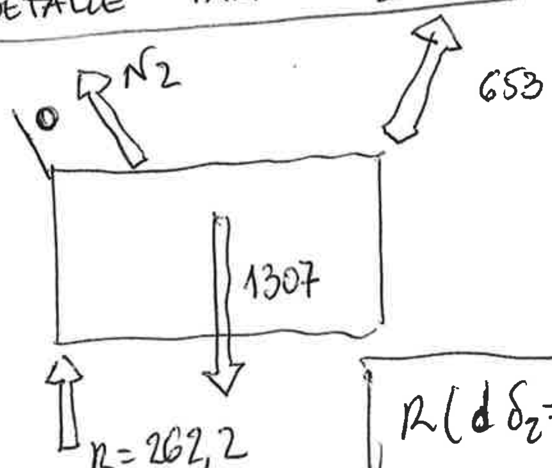
$\sum F_H \neq 0$

el equilibrio sería imposible.

LA CUENTA DE LA VIEJA.



UN ANÁLIS DE DETALLE PARA $d\delta_2=0$

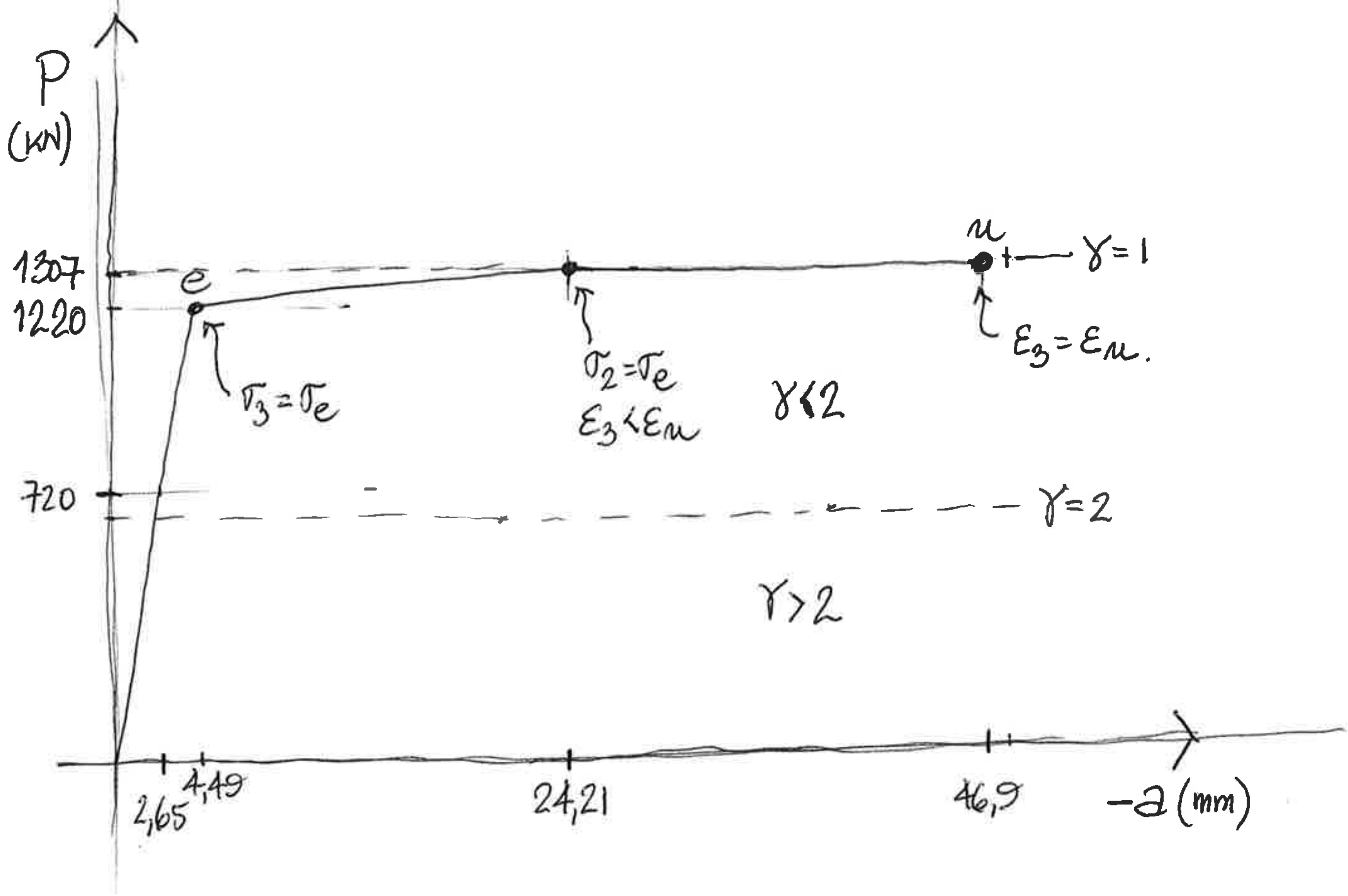


$\sum F_H = 0 \Rightarrow N_2 = 653$

$\sum F_V = 0 \Rightarrow R = 262{,}2$

$$\boxed{R(d\delta_2=0) \equiv R(d\delta_1=0)}$$

SOLIDO DEFORMABLE (II). EJEMPLO COMPLICADO

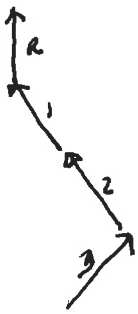


$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_3\end{Bmatrix} = \begin{bmatrix} 0 & -\frac{4}{3}m\times\frac{3}{5} & \frac{16}{3}m\cdot\frac{3}{5} \\ 1m\cdot\frac{4}{5} & 0 & \cancel{\frac{20}{3}m\cdot\frac{3}{5}} \\ 4m\cdot\frac{4}{5} & 4m & 0 \end{bmatrix}\begin{Bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{Bmatrix}$$

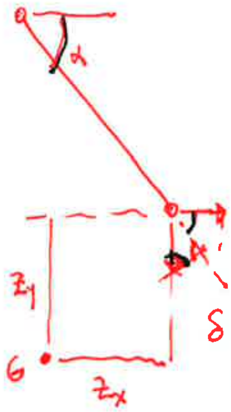
$g = 2$

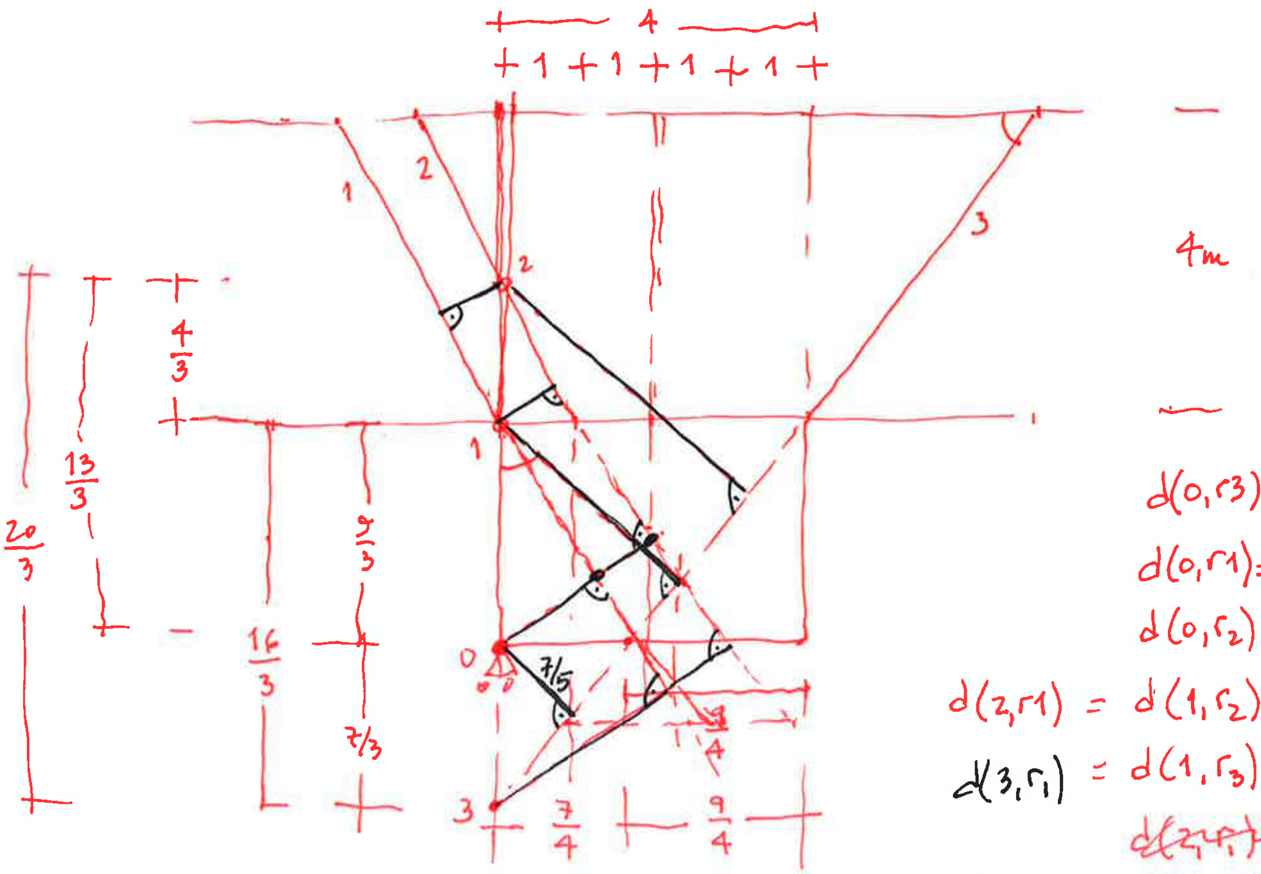
$s = 3$

0	−0,8m	3,20m
0,8m	0	4m
3,20m	4m.	0



$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_3\end{Bmatrix} = \begin{bmatrix} 0 & -0{,}8m & 3{,}20m & 1{,}8m \\ 0{,}8m & 0 & 4m & 2{,}60m \\ 3{,}20m & 4m & 0 & 1{,}40m \end{bmatrix}\begin{Bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{Bmatrix}$$





$d(0, r3) = \frac{7}{5} = 1{,}20$

$d(0, r1) = \frac{9}{5} = 1{,}8$

$d(0, r_2) = \frac{13}{5} = 2{,}60$

$d(2, r1) = d(1, r_2) = \frac{4}{5} = 0{,}8m.$

$d(3, r_1) = d(1, r_3) = \frac{16}{5} = 3{,}20m$

~~$d(2, r_1) = \frac{4}{5} = 1$~~

$d(3, r_2) = d(2, r_3) = \frac{20}{5} = 4m.$

$d(3, r_1)$