Diseño de vigas resistentes para un problema de flexión simple, con comprobación inicial de rigidez

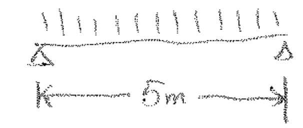
La madera empleada resiste con seguridad tensiones normales de 10 N/mm^2 y tangenciales de 1 N/mm^2; su módulo de Young es de 12,5 kN/mm^2.

El acero empleado resiste con seguridad tensiones normales de 180 N/mm^2 y tangenciales de 100 N/mm^2; su módulo de Young es de 210 kN/mm^2.

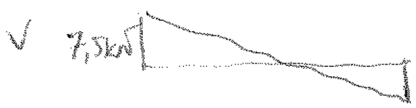
Como requisito de rigidez se considera como tolerables distorsiones medias no mayores de 4 mm/m.

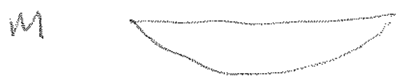
En el último ejemplo (pp. 7 y 8) se ofrecen distintas soluciones con tablones estándar, inseguras, ajustadas o exageradas (desde el punto de vista de la resistencia).

En todos los casos se sugiere como determinar la variación de la tensión tangencial en una sección.



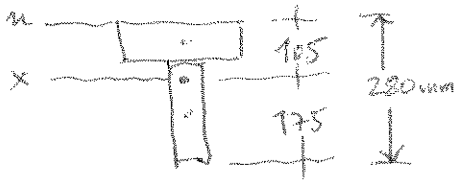
carga ligera de una vigueta de cubierta... separación $\simeq 700mm$





$$M_{max} = \frac{pl^2}{8} = 9{,}38 \text{ mkN.}$$

VIGA DE MADERA EN T.   tablones $210 \times 70\ mm^2$



$$A = 29.400\ mm^2$$

$$P_{u=0} = 3.087\ mm^2 \cdot m$$

$$u_g = \frac{P_{u=0}}{A} = 105\ mm$$

$$I_x = \frac{1}{12}\, 70mm \times 210mm\, (0{,}07m)^2 +$$

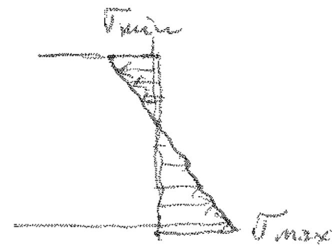
$$+ \frac{1}{12}\, 70mm \times 210mm\, (0{,}21m)^2 +$$

$$+ 2\,(70mm \times 210mm) * (0{,}07m)^2 =$$

$$= 14700\ mm^2\,(0{,}00408\ m^2 + 0{,}0098\ m^2) =$$

$$= 204\ mm^2 \cdot m^2$$

$W_{sup} = 1944\ mm^2 \cdot m$ $W_{inf} = 1.166\ mm^2 \cdot m \equiv W_{min}$



$$\sigma_{max} = \sigma_{inf} = \frac{9{,}38\ mkN}{1166\ mm^2 \cdot m} = 8{,}04\ \frac{N}{mm^2} < 10\ N/mm^2$$

$\sigma_{min} = 4{,}83\ \frac{N}{mm^2}$ → curvatura

$M_f = 10N/mm^2 \cdot 1166\ mm^2 \cdot m = 11{,}7$ mkN.

---

brazo de palanca: $S_i = 175mm \times 70mm \times \frac{175mm}{2} = 1072\ mm^2 \cdot m$

$$z = \frac{I}{S_i} = 0{,}19m = 190\ mm \qquad \frac{z}{h} = 0{,}68 \simeq \frac{2}{3}$$

$V_f : A_R \times f_{mv}$ $A_R = 70\ mm \times 190\ mm = 13322\ mm^2$

$$V_f = A_R \times 1N/mm^2 = 13{,}3\ kN > V_{max} = 7{,}5kN$$

VIGA DE MADERA. DISTRIBUCIÓN τ

$V = 7,5\ kN$

$\frac{V}{I} = \cancel{0,0\ldots} = 36,8 \times 10^{-3}\ \frac{kN}{mm^2 \cdot m^2} = 36,8\ \frac{N}{mm^2 \cdot m^2}$

$y\,b(y)$

$y\,b(y)$ $\times 10^3\ mm^2$

$-22,1$

$-7,35 \,||\, -2,45$

$12,3$

$\frac{\partial \tau b}{\partial y} = \frac{V}{I}\, y\,b(y)$ $\times 10^{-3}\ N/mm^2$

-813

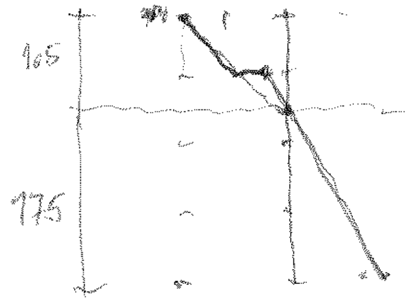
$-270 \,||\, -90,1$

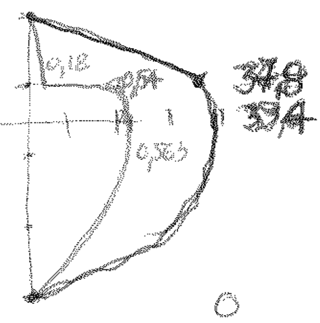
-270

452

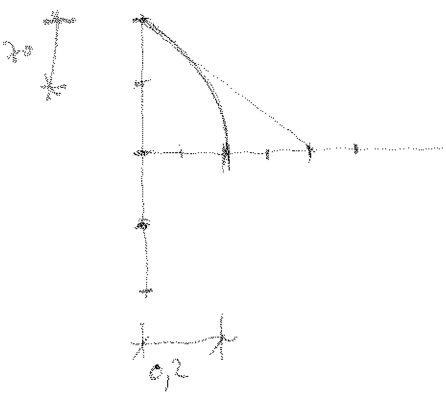
$\tau b\ (\times 10^3\ \frac{N}{m})$

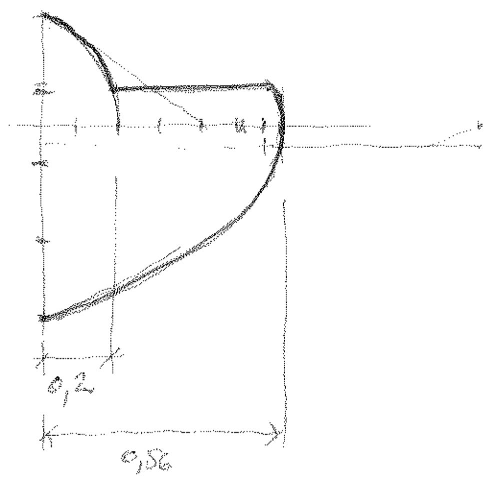
$\tau\ (N/mm^2)$





MÉTODO DE "REDONDEO"





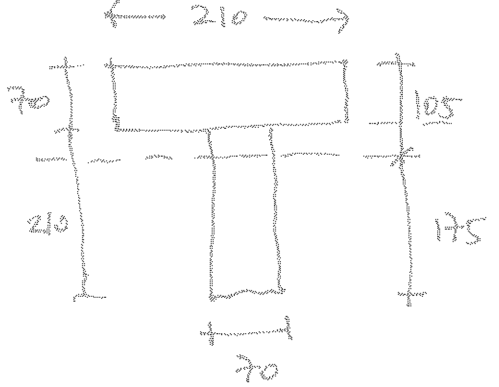
$\dot{c}_{max} = \frac{9,38\ mkN}{12,5\ kN/mm^2 \times 204\ mm^2 \cdot m^2} = 3,68 \times 10^{-3}\ m^{-1} = 3,68\ mrad/m$ $\qquad \delta_{tol} = 10\ mm.$

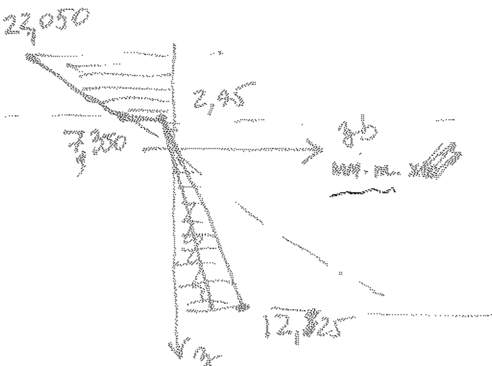
$\lambda = 17,9$ $\quad \varepsilon_{max} = 0,643\ mm/m.$ $\quad \varepsilon_{min} = 0,386\ mm/m$ $\quad \bar{\varepsilon} = 0,514\ \frac{mm}{m}$ $\quad \left(\frac{\delta}{\ell}\right)_{tol} = 2\ mm/m$

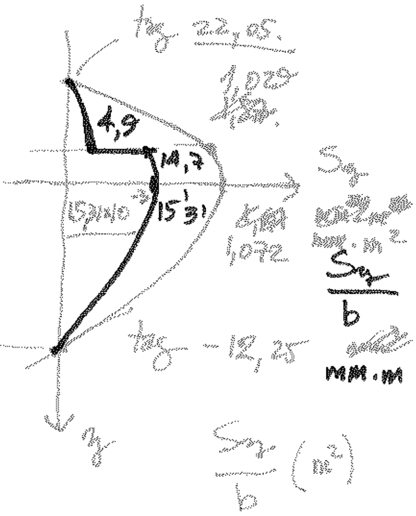
$\phi_{tol} = 4\ mm/m$

$\frac{\delta}{\ell} = \frac{5}{48} \times 3,68 \times 10^{-3}\ m^{-1} \times 5\ m = 1,92\ mm/m$ $\quad$ aceptable

$\lambda = 17,9 \qquad \lambda' = 12 \times \frac{0,8\ mm/m}{0,514\ mm/m} = 18,7 \quad \lambda < \lambda'$ OK.







$$\frac{V(x)}{I} = \frac{7{,}5\,kN}{204\,mm^2m^2} = 36{,}8\times10^{-3}\,\frac{kN}{mm^2\cdot m^2} =$$

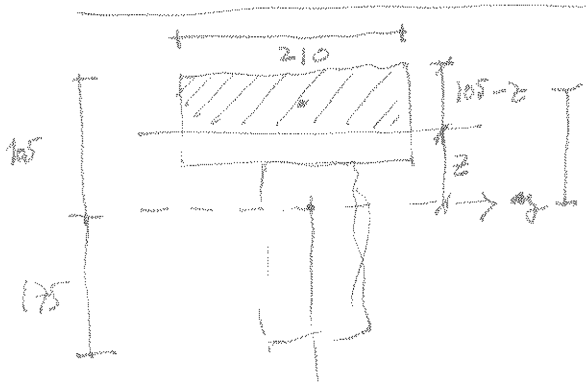
$$\tau_{max} = 36{,}8\times10^{-3}\,\frac{kN}{mm^2m^2}\times 15{,}31\,mm^2\,m/m\cdot m$$

$$= \cancel{36{,}8\times10^{-3}\,\frac{kN}{mm\cdot m}}\ \cancel{563\times10^{-3}\,kN}$$

$$= 563\times10^{-3}\,\frac{kN}{mm\cdot m} = 0{,}563\,\frac{N}{mm^2}$$

$$\tau_{ala/alma} = 36{,}8\times10^{-3}\,\frac{kN}{mm^2\cdot m^2}\times 14{,}7\,mm\cdot m$$

$$= 0{,}54\,\frac{N}{mm^2}$$



$S_y(z) = (z-(-105))\cdot\frac{1}{2}(-105+z)\cdot 210$

$b(z) = 210$

$z<-35$: $\tau = \frac{V(x)}{I_y}\cdot\frac{S_y(z)}{b(z)} = \frac{V(x)}{I_y}\cdot\left[\frac{1}{2}(z+105)(z-105)\right]$

$z>-35$:

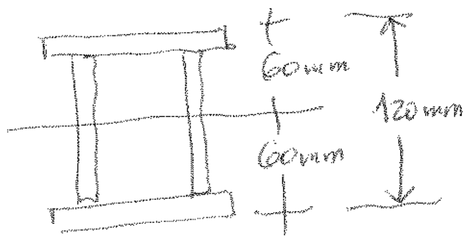
$S_y(z) = (175-z)\cdot\frac{1}{2}(175+z)\cdot 70$

$b_y(z) = 70$

$z>-35$: $\tau = \frac{V(x)}{I_y}\cdot\left[\frac{1}{2}(175-z)(175+z)\right]$

VIGA CUADRADA HUECA EN ACERO.

pletinas $\neq 100 \times 10\ mm^2$



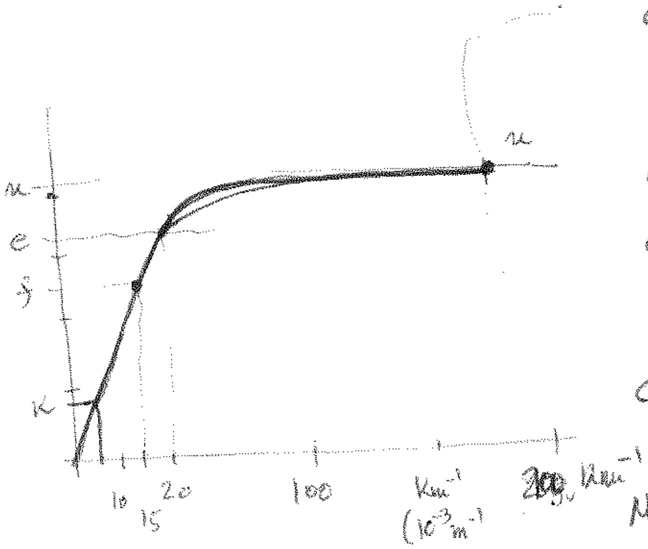
$A = 4000\ mm^2$

$I = 7{,}73\ mm^2 \cdot m^2$

$W_{sup} = W_{inf} = W_{min} = 129\ mm^2 \cdot m.$

$$\sigma_{max} = \frac{9{,}38\ mkN}{129\ mm^2 \cdot m} = 72{,}7\ N/mm^2 \leq f_s = \boxed{180\ N/mm^2}$$

$$c = \frac{9{,}38\ mkN}{210\ kN/mm^2 \times 7{,}73\ mm^2 \cdot m} = 5{,}78 \times 10^{-3}\ m^{-1} = \frac{1}{173\ m}$$



$\boxed{\sigma_e = 260\ N/mm^2}$

$\varepsilon_e = 1{,}24\ mm/m$

$$M_e = W \sigma_e = 33{,}5\ mkN$$

$$c_e = \frac{\varepsilon_e}{h/2} = 20{,}6 \times 10^{-3}\ m^{-1} = \frac{1}{48{,}5}$$

$$c_u = \frac{10\ mm/m}{60\ mm} = 0{,}17\ m^{-1} = \frac{1}{6m}$$

$$M_u \simeq \sigma_e \cdot 2 \cdot S_{1/2} = \sigma_e \cdot W_p$$

$$S_{1/2} = 1000\ mm^2 \times 55\ mm + 2 \times 500\ mm^2 \times 25\ mm = 80\ mm^2 \cdot m$$

$$M_u \simeq 41{,}6\ mkN$$

$$M_f \simeq \frac{41{,}6\ mkN}{1{,}44 \times 1{,}12} = 25{,}71\ mkN \gtrsim \frac{M_e}{1{,}44}$$

$$\frac{W_p}{W} = 1{,}24.$$

$\boxed{\sigma_{max}(M_f) > f}$ comentario

$$\boxed{c_f = 15{,}84 \times 10^{-3}\ m^{-1}} \leq c_e$$

$$z = \frac{I}{S_{1/2}} \simeq 0{,}10\ m \approx 96\ mm \quad 96{,}6\ mm.$$

$$\frac{z}{h} \simeq 0{,}8$$

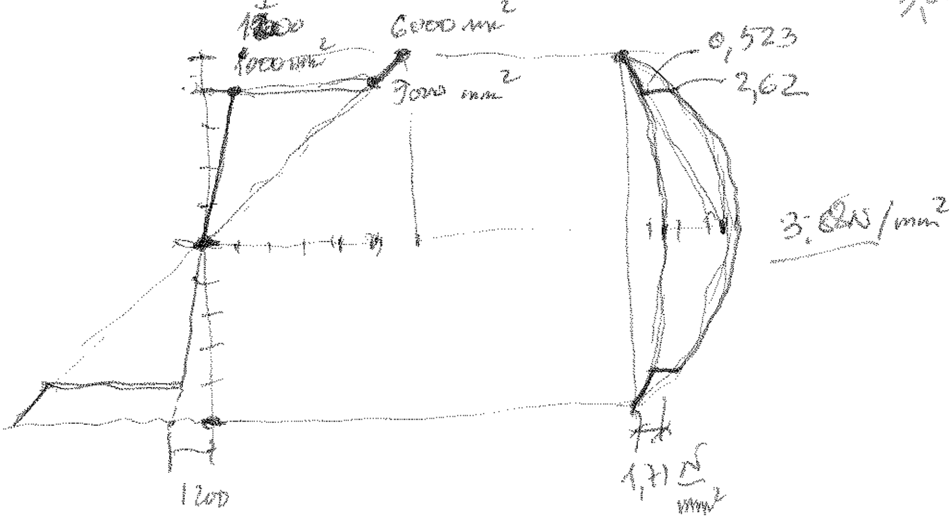
$$\frac{h}{z} = 1{,}24.$$

VIGA CUADRADA HUECA DE ACERO.

$A_e > 96\,mm \times 2 \times 10\,mm = 1920\,mm^2 \to 1932\,mm^2$

$V_f = 1920\,mm^2 \times 0{,}1\,kN/mm^2 = 192\,kN > V_{max} = 7{,}5\,kN$
193 kN

Diagrama. $\frac{V}{I} = 0{,}970 \frac{N}{mm^2 \cdot m^2}$ $\tau_{max} = \frac{V}{A_e} = 3{,}91 \frac{N}{mm^2}$ 3,88 revisar.



$C_{max} = \frac{9{,}38\,m\,kN}{210\,kN/mm^2 \times 7{,}73\,mm^2\,m^2} = 5{,}78 \times 10^{-3}\,m^{-1}$

$\lambda = 41{,}7$ $\varepsilon = 0{,}346\,mm/m.$

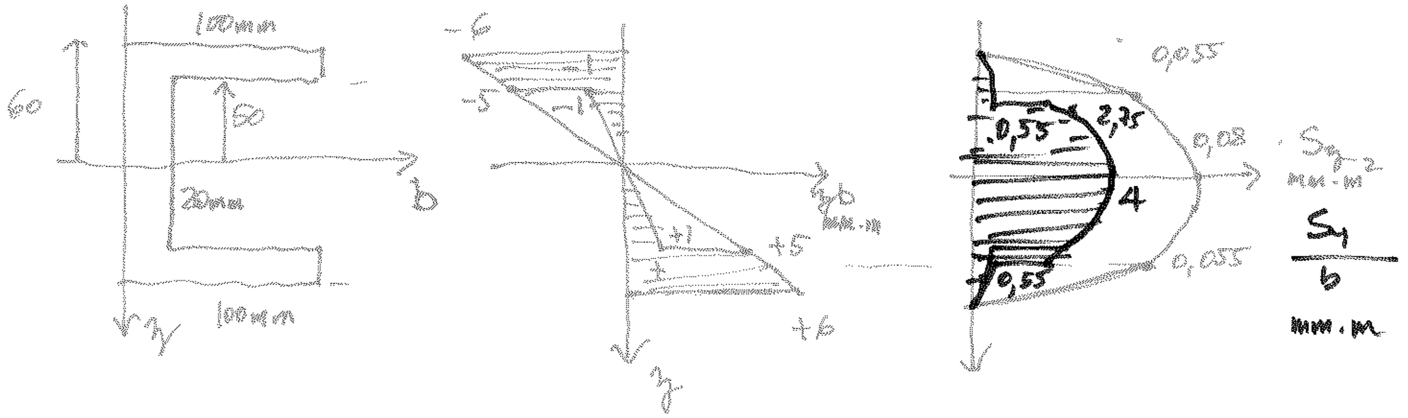
$\frac{\delta}{\ell} = 5{,}78 \times 10^{-3}\,m^{-1} \times \frac{5}{48} \times 5\,m = 3{,}01\,mm/m$ INTOLERABLE.

el diseño debe ser cambiado y el proceso de comprobación ~~vuelto~~ repetido...

~~a canto constante~~ la inercia debería ser 1,5 veces mayor.

~~a igualdad inercia, el canto debería ser 1,5 veces mayor~~

$\lambda' = 12 \times \frac{0{,}8}{0{,}346} = 27{,}7$ → conservando el módulo resistente, el canto debería ser 1,5 veces mayor.



$$\frac{V}{I} = 970 \frac{N}{mm^2 \cdot m^2}$$

$$\tau_{max} = 970 \frac{N}{mm^2 \cdot m^2} \times 4 \, mm \cdot m = 3{,}88 \frac{N}{mm^2}$$

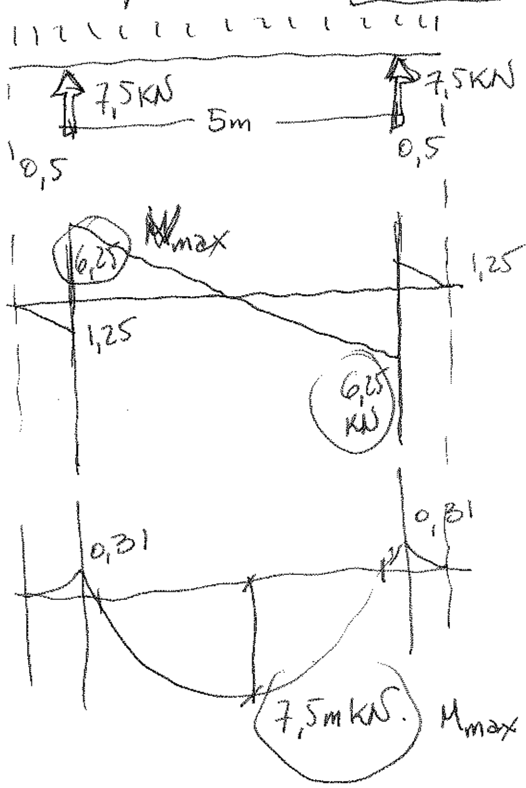
$$\tau_{ala/alma} = 970 \frac{N}{mm^2 \cdot m^2} \times 2{,}75 \, mm \cdot m = 2{,}67 \frac{N}{mm^2}$$

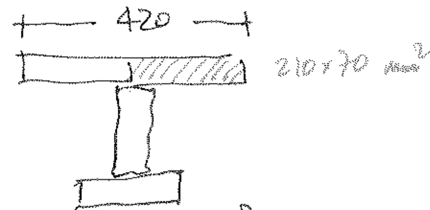
U.P.M. | Asignatura | 2002 año | mes | día

E.T.S.A.M. | 1er apellido | 2º apellido | Nombre | Nº expediente

10 N/mm² 1N/mm².

p = 6 kN/m² = $\boxed{2,5\ kN/m}$





I = 720 mm². m.

W_{min} = 3.428 mm². m.

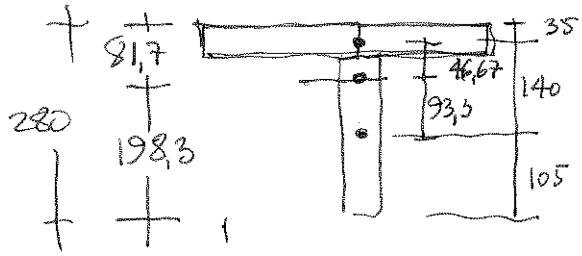
$S_{1/2}$ = 3.259 mm². m.

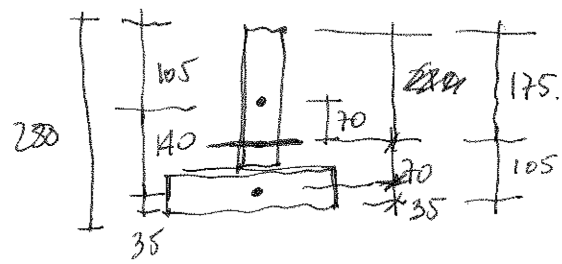
Z = 0,22m = 0,63h.

A_R = 15.467 mm².

R_M = 34,28 mkN

R_V = 15,47 mkN





W_{min} = 514,5 mm². m.

R_M = 5,15 mkN

A_R = 9.800 mm².

R_V = 9,8 KN.

I_g = 190,5 + 54 + 12 = 256 mm². m²

W_{min} = 1.293, mm². m

$S_{1/2}$ = 1376 mm² m.

Z = 0,19m = 0,67h = 186mm

A_R = 13.044 mm²

R_M = 12,9 m kN.

R_V = 13,84kN

I_g = 144 + 54 + 6 = = 204 mm². m²

W_{min} = 1.166,2 mm². m

$S_{1/2}$ = 1.072 mm². m

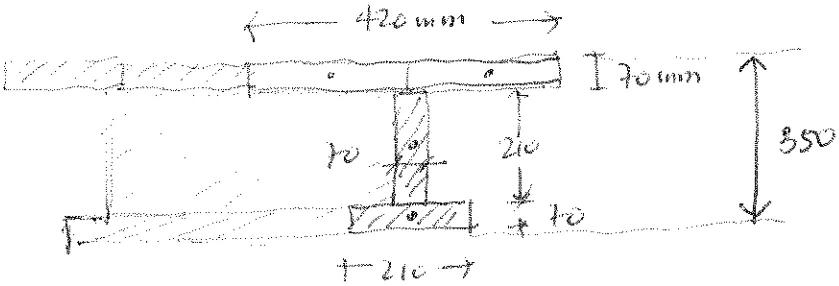
Z = 0,19m = 0,68h

A_R = 13.322 mm²

R_M = 11,66 kN. m.

R_V = 13,3 KN

FLEXIÓN. CORTANTE. UN EJEMPLO DE DISTRIBUCIÓN DE TENSIONES   Abril 2001   EJEMPLOS 8



$W_{min} = 3.428\ mm^2 \cdot m.$

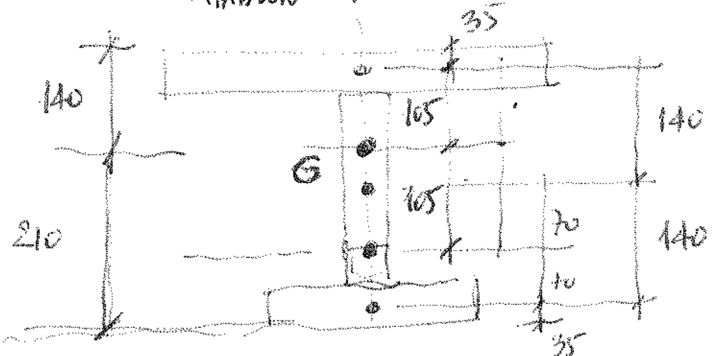
$A_{TABLÓN} = 14.700\ mm^2$

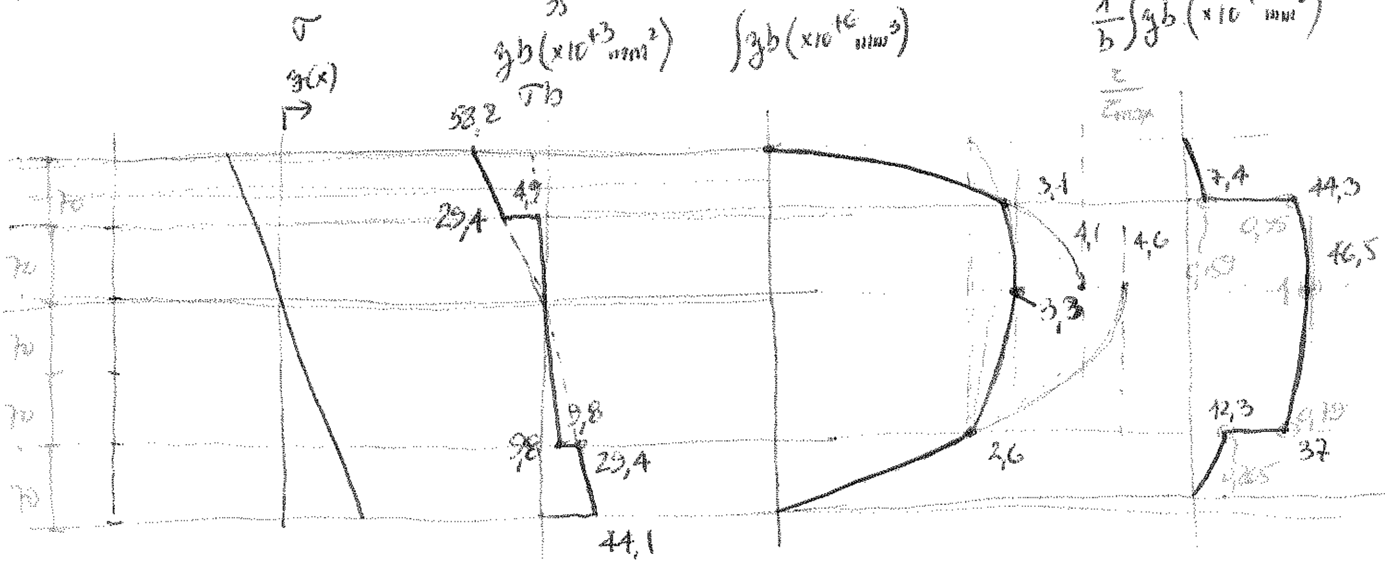
VS

$I_g = (648 + 12 + 6 + 54)\ mm^2 \cdot m^2 = \underline{720}\ mm^2\, m^2$

$S_{1/2} = 3259\ mm^2 \cdot m$

$z = 0,22\ m = 0,63h$





$$\tau = \frac{N_y}{N/B} V \times \left(\frac{1}{b}\int yb\right) \times \frac{1}{I}$$

$V_f = 70\,mm \times 221\,mm \times 1\,N/mm^2 = 15,47\ kN.$