Funicularity and Equilibrium for high-performance conceptual structural design

Leonardo Todisco
lt@he-upm.com
Research Group in Structural Concrete
School of Civil Engineering
Polytecnic University of Madrid, Spain

The concept of funicularity

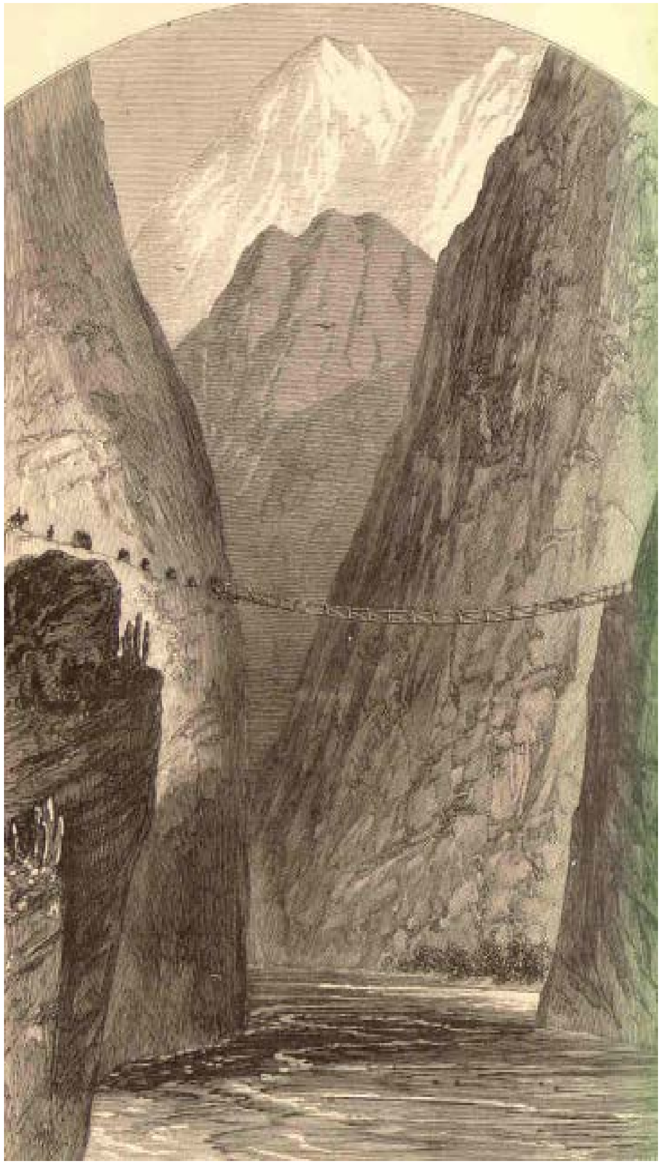
Computational tools for Conceptual Design

Arch bridges with curved deck

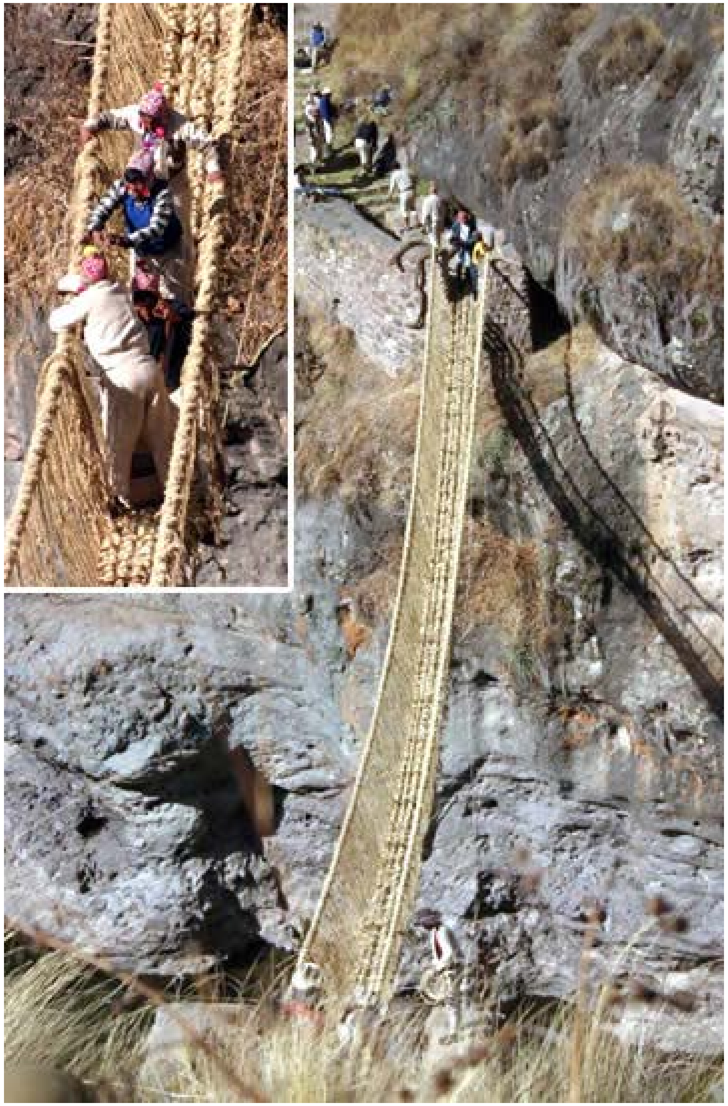
Funicular post-tensioned structures

Conclusions and future researches

'funicular': from the Latin word *funiculus*, diminutive of *funis*, meaning 'slender rope'.



Inca suspended bridge 1300 AD (Squier 1877)



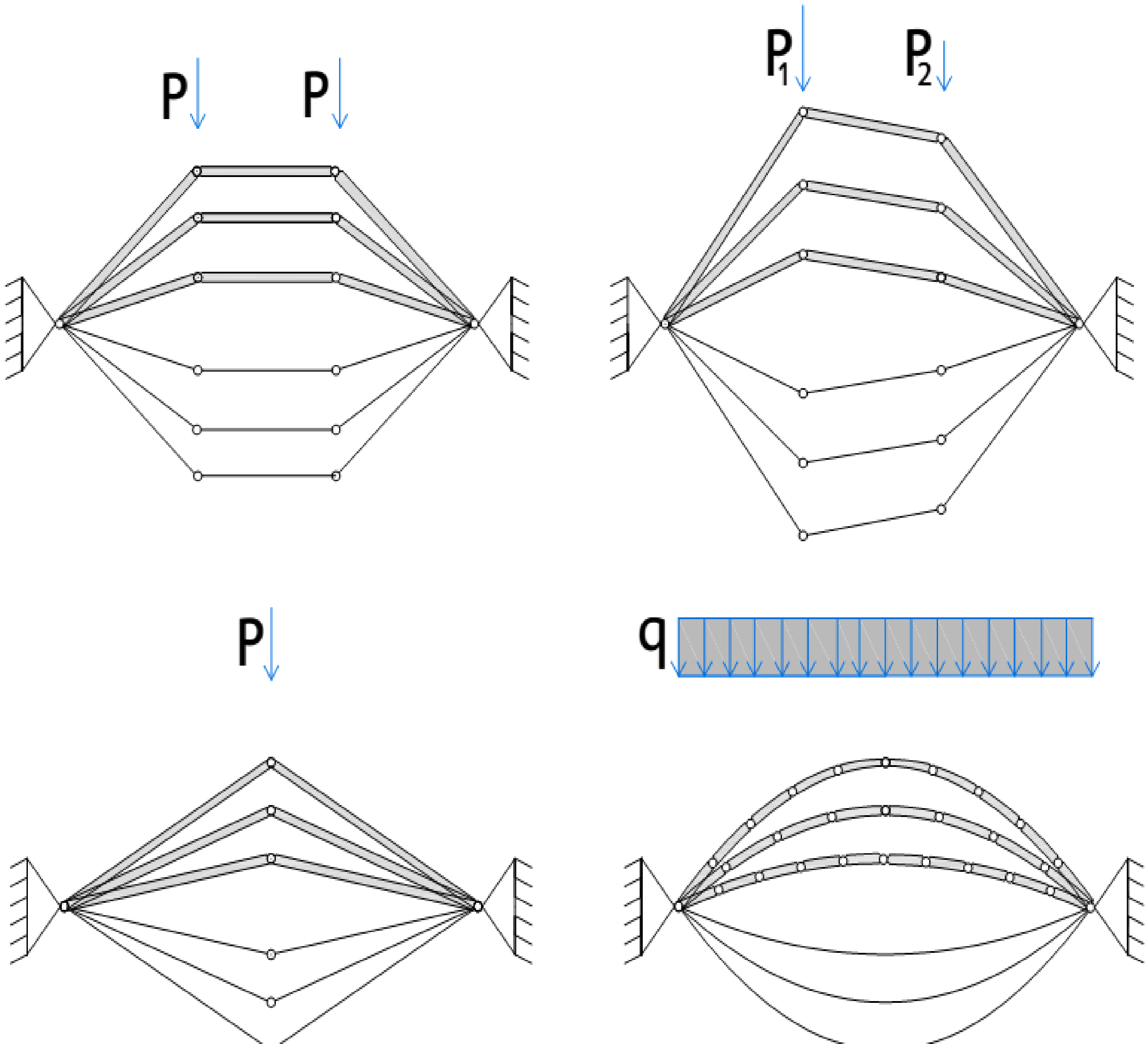

Q'eswachaca Bridge, Cusco, Perú (Wikiperia)

Palma de Mallorca Cathedral 1532

Trulli in Apulia, Italy (Wikipedia)

Image taken from Apuliabase.com

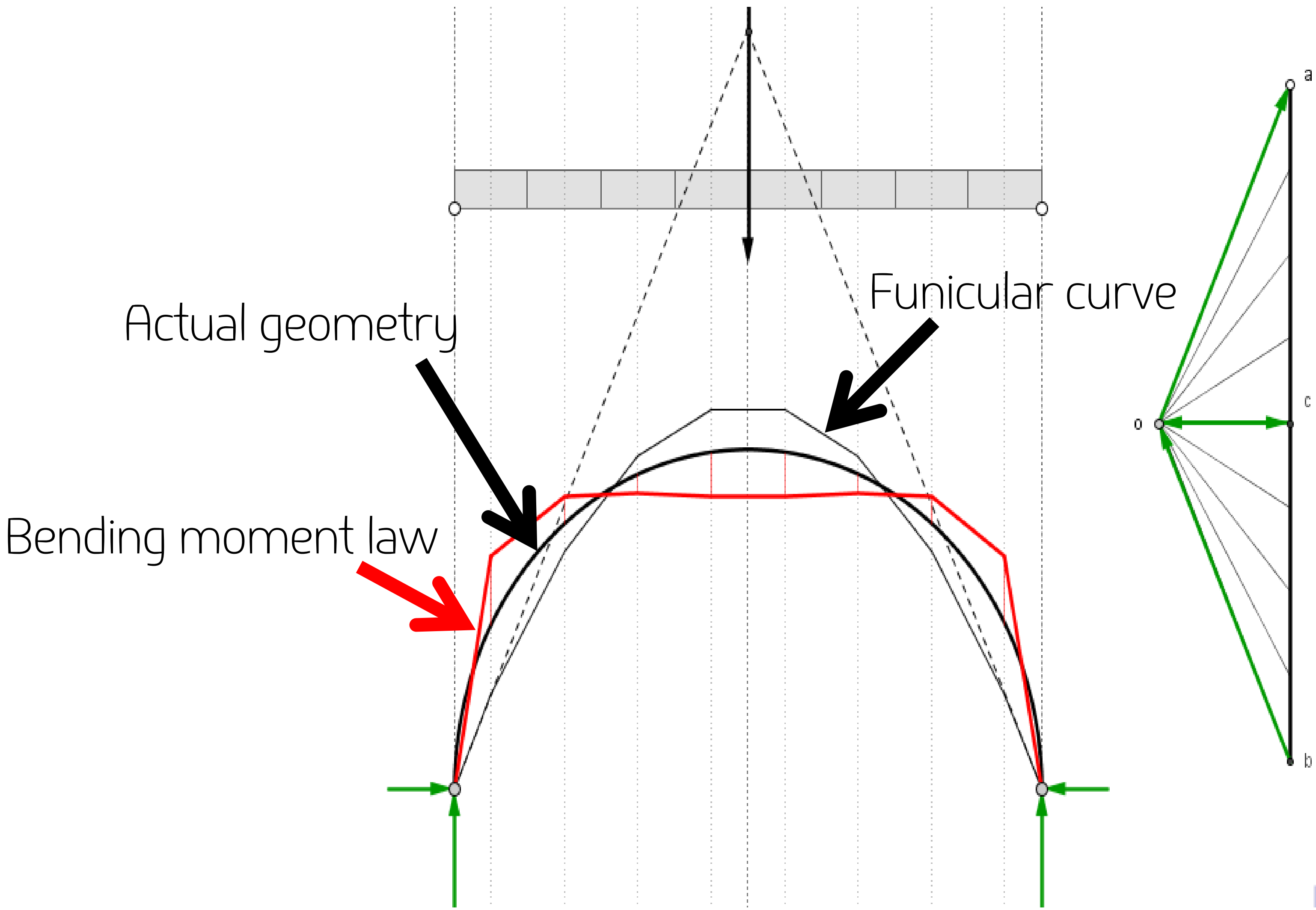
Apuliabase


P
P
P_1
P_2
P
q

Akashi Kaikyō Bridge, Japan

Gateway Arch (Saarinen) in St. Luis Missouri, USA

Speccheri Dam (TN), Italy


Actual geometry
Funicular curve
Bending moment law
a
c
o
b

Method of moments: bending moments in each point of the curve have to be zero; the funicular configuration corresponds to the moment diagram of a beam with the same span and distribution of loads

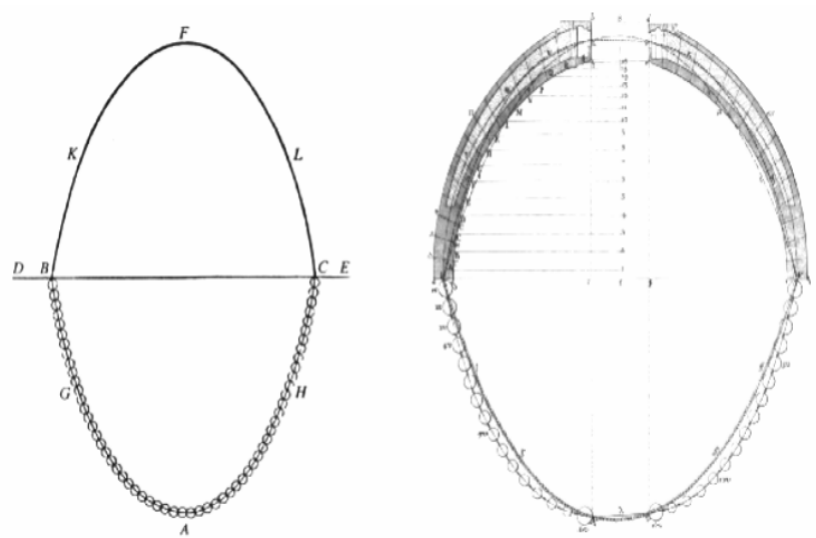
$$M(x) = M_o(x) - Hy(x) \qquad y(x) = \frac{M_o(x)}{H}$$

Numerical methods: Numerical methods have been developed in order to find the solution to a problem characterized by a strong geometric non-linearity. They are numerical algorithms where an iterative process stops when a static equilibrium is reached .

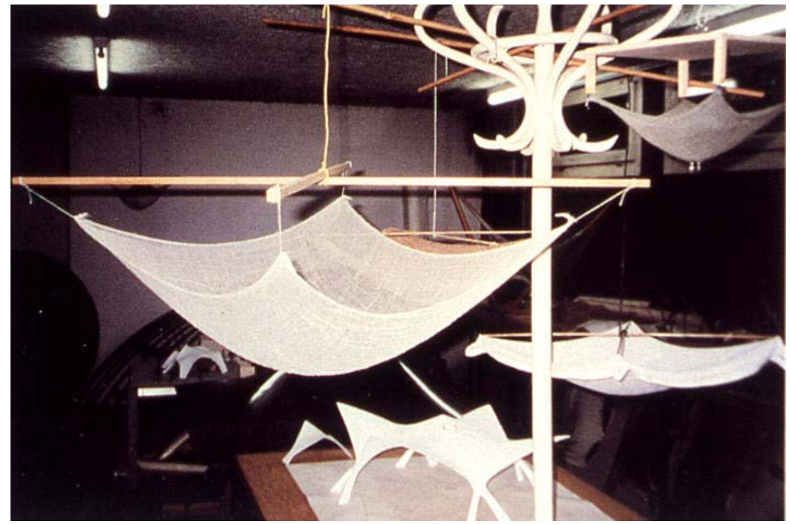
Particle-spring systems: Conceptually this methodology is similar to the physical models with hanging elements. The stiffness of the springs corresponds to the axial stiffness of the material used with reduced physical models.

Physical models: physical models are models in which a set of loads is suspended to a network of wires.

Inverting and 'freezing' the system it is possible to obtain compression-only structures



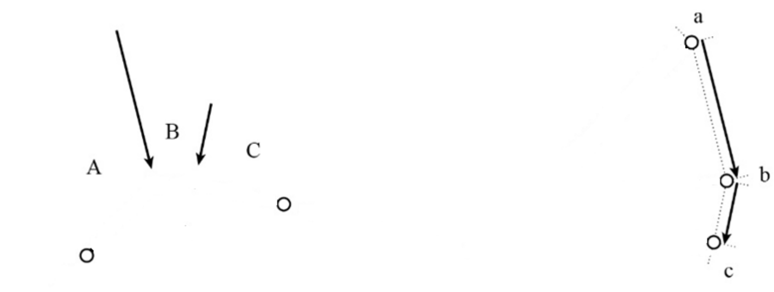
Hanging chain and correspondent inverted arch for the structural safety assessment of San Peter dome in Rome (1743)



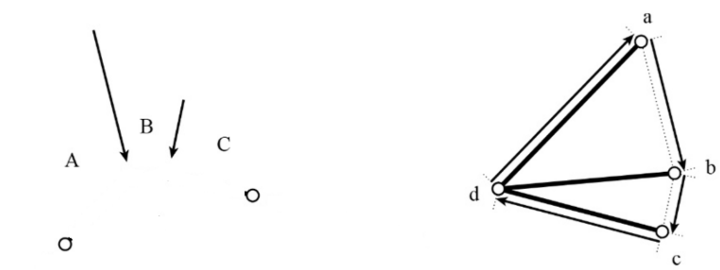
Physical models by Isler (1926-2009)

Force Frozen at MIT, January 2015

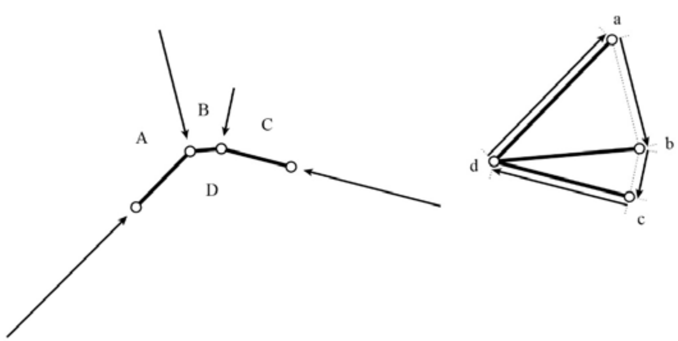
Graphic statics: using the reciprocal relationship between funicular polygon and force polygon is possible to find compression-only and tensile-only geometries



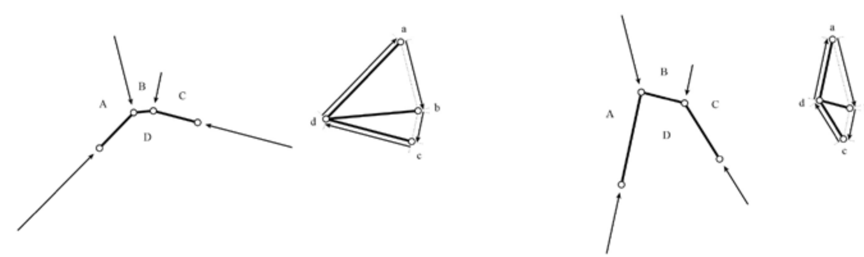
1



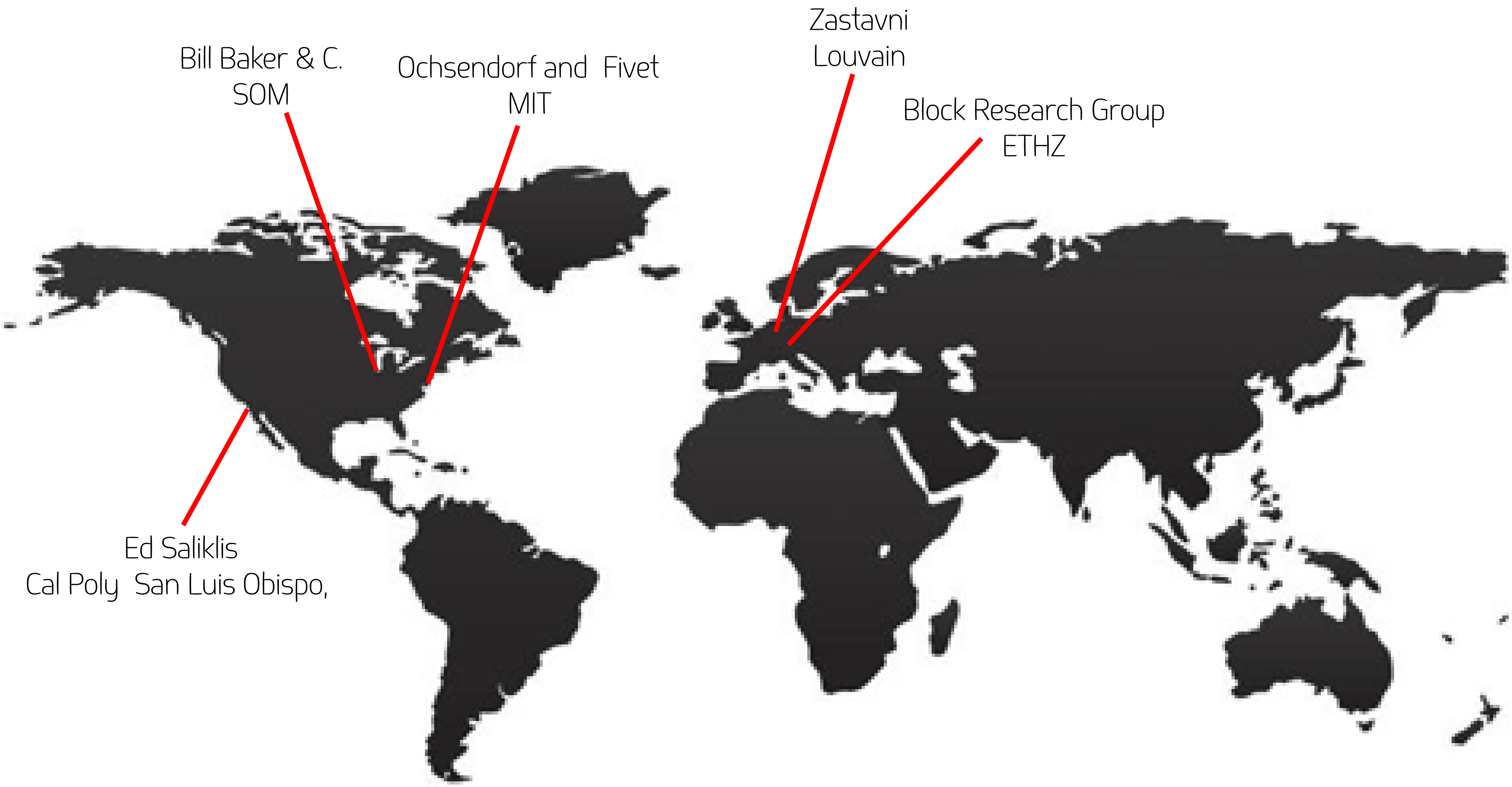
2



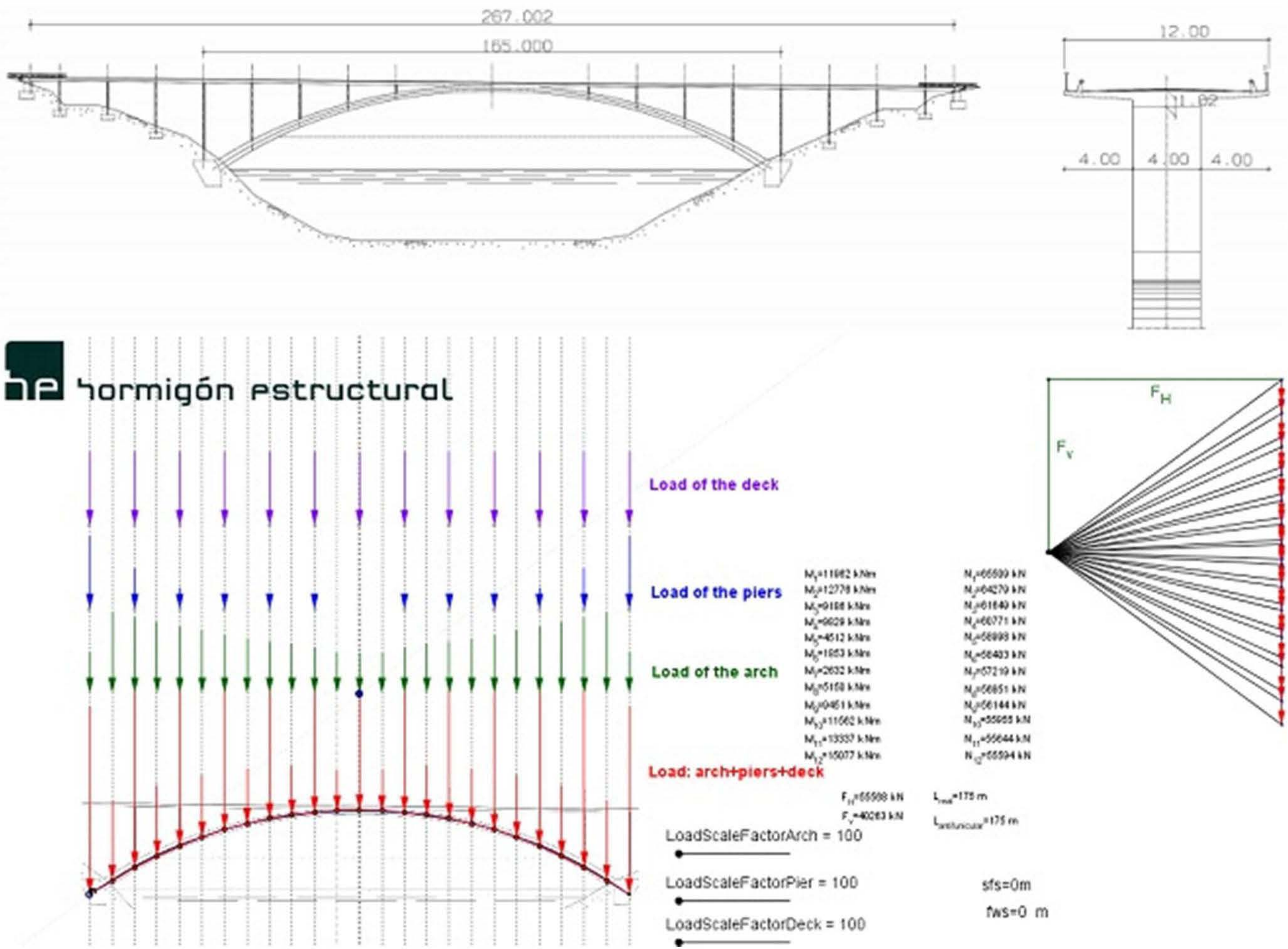
3



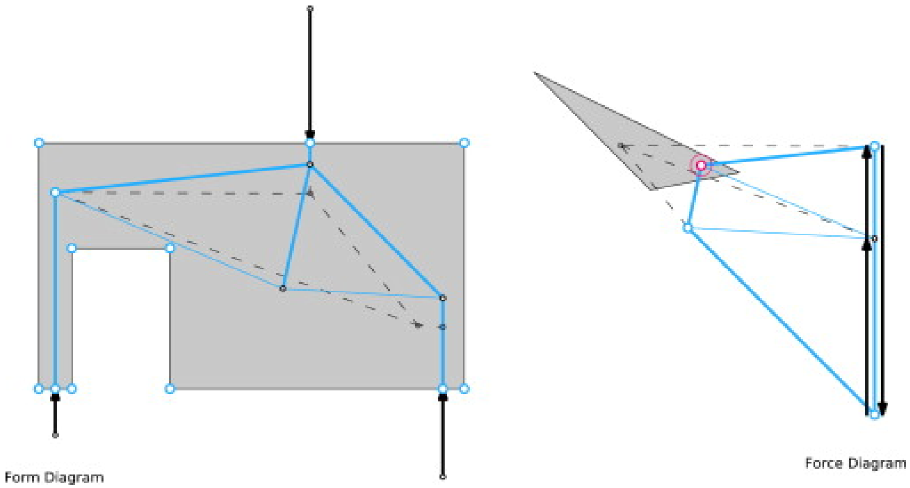
4



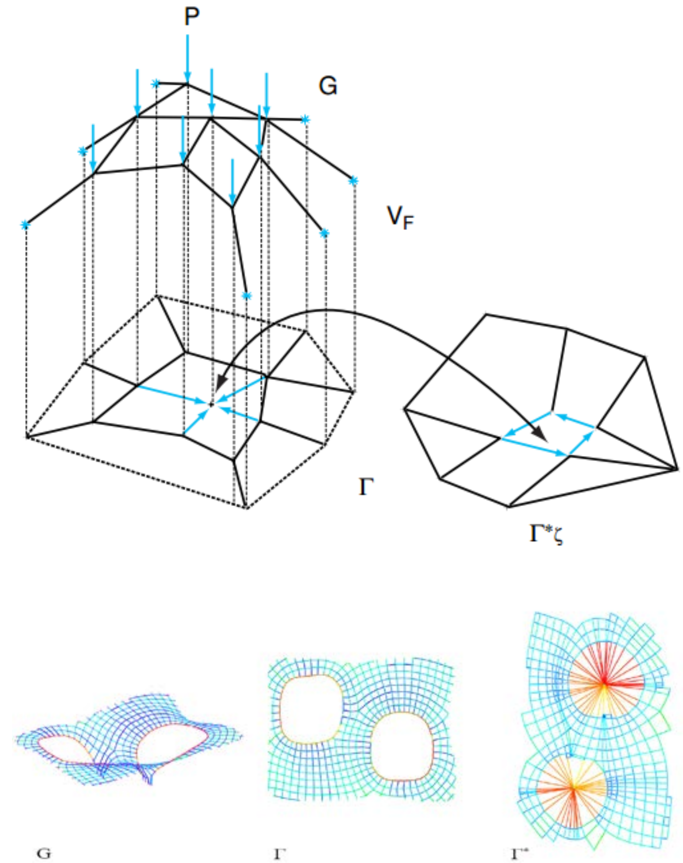
Bill Baker & C.
SOM
Ochsendorf and Fivet
MIT
Zastavni
Louvain
Block Research Group
ETHZ
Ed Saliklis
Cal Poly San Luis Obispo,



(Todisco 2014)



Constraint-based structural equilibrium design
(Fivet and Zastavni 2012).



Thrust Network Analysis.
Taken from Block (Block 2009)

Graphic Statics

References:

- *Form and Forces: Designing Efficient, Expressive Structures*; E. Allen , W. Zalewski

Web resources:

- http://web.mit.edu/masonry/
- http://www.block.arch.ethz.ch/equilibrium/
- http://acg.media.mit.edu/people/simong/statics/data/
- http://n.ethz.ch/~lballo/index.php?id=6614 (tutorial 9:38 min)
- http://sites.uclouvain.be/structech_loci/

The concept of funicularity

Computational tools for Conceptual Design

Arch bridges with curved deck

Funicular post-tensioned structures

Conclusions and future researches

Graphic Static tool:

- Active statics (MIT)
- eQUILIBRIUM (ETHZ)
- RhinoStatics (MIT)
- Geogebra

Form-finding tool:

- CADenary (MIT)
- RhinoVAULT (ETHZ)
- FormFinding WebApp (Princeton)

Optimization tool:

- structureFIT (MIT)
- ESO, BESO (Melbourne)

"By linking **parametric design** to **structural analysis** and **optimization**, architects and structural engineers can explore design in the conceptual design phase through informed geometry alterations…"

Strong need for this kind of tools, devoted to design, to explore and not only to analyze structures.

A combination of different software in a parametric, three-dimensional, integrate and interactive environment.

Rhinoceros® NURBS modeling for Windows

grasshopper GENERATIVE MODELING FOR RHINO

Karamba PARAMETRIC STRUCTURAL MODELING

karambaToSofistik plug-in → SOFiSTiK

The concept of funicularity

Computational Tools for conceptual design

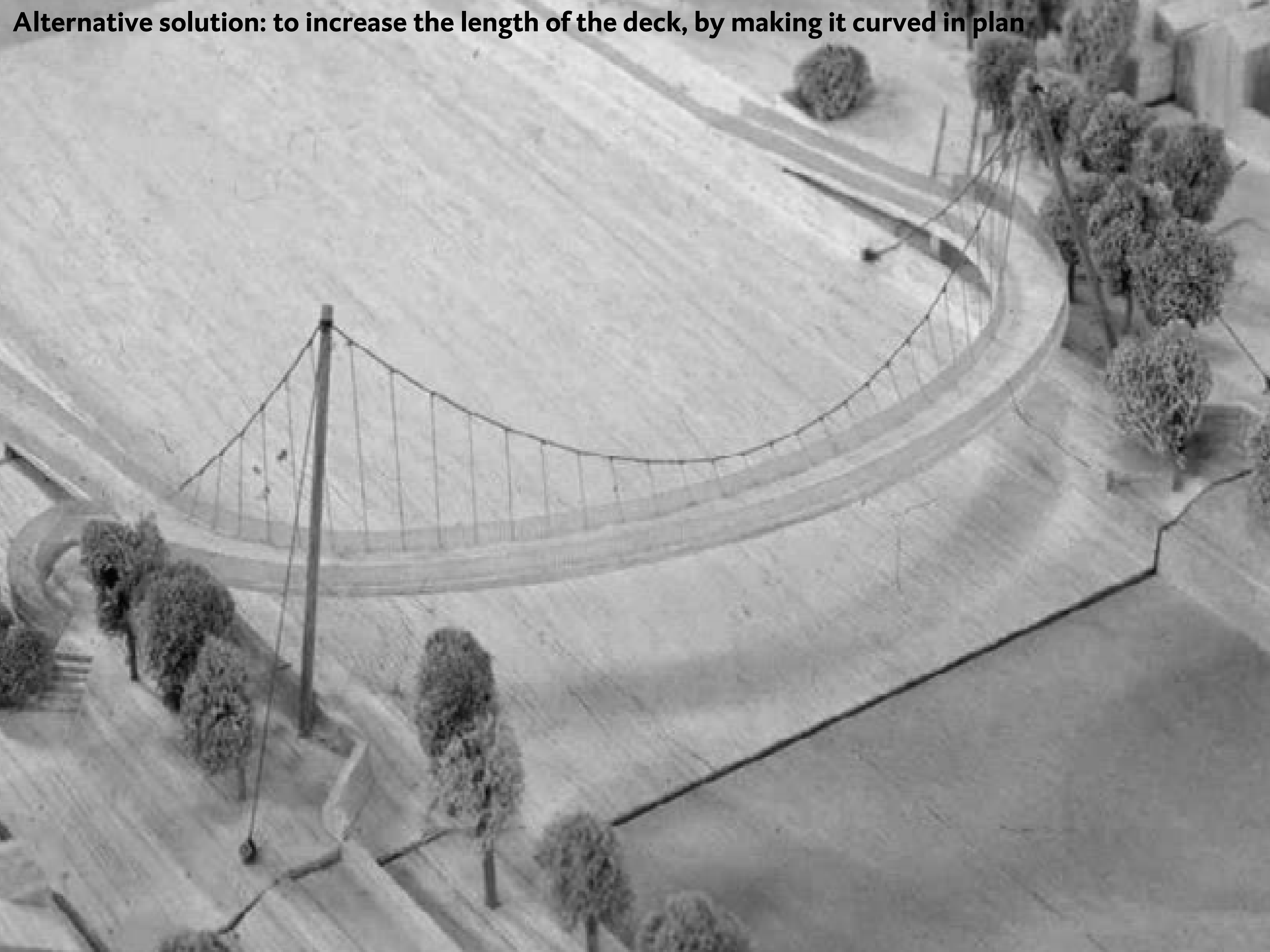
Arch bridges with curved deck

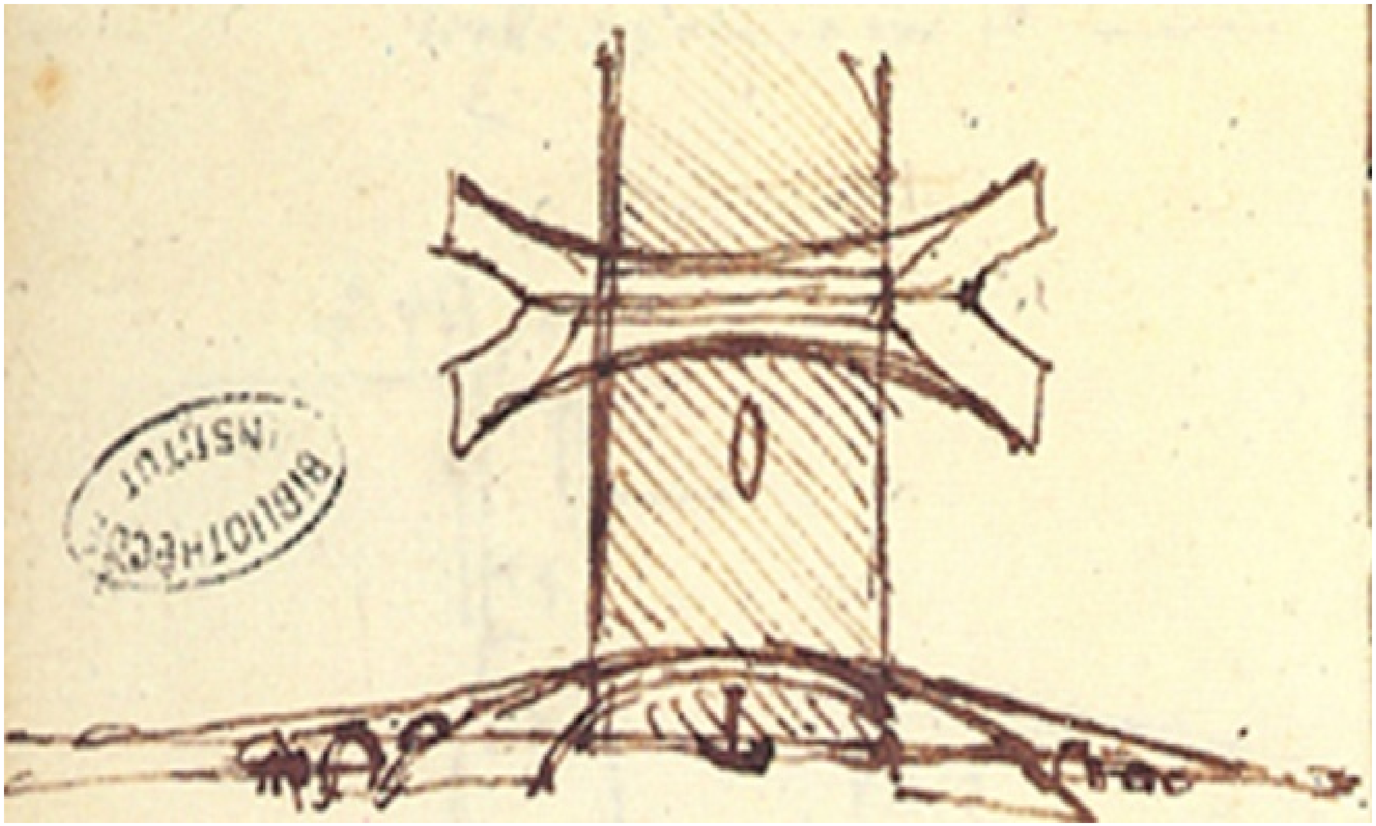
Funicular post-tensioned structures

Conclusions and future researches

Common solution: to incorporate external ramps

Alternative solution: to increase the length of the deck, by making it curved in plan





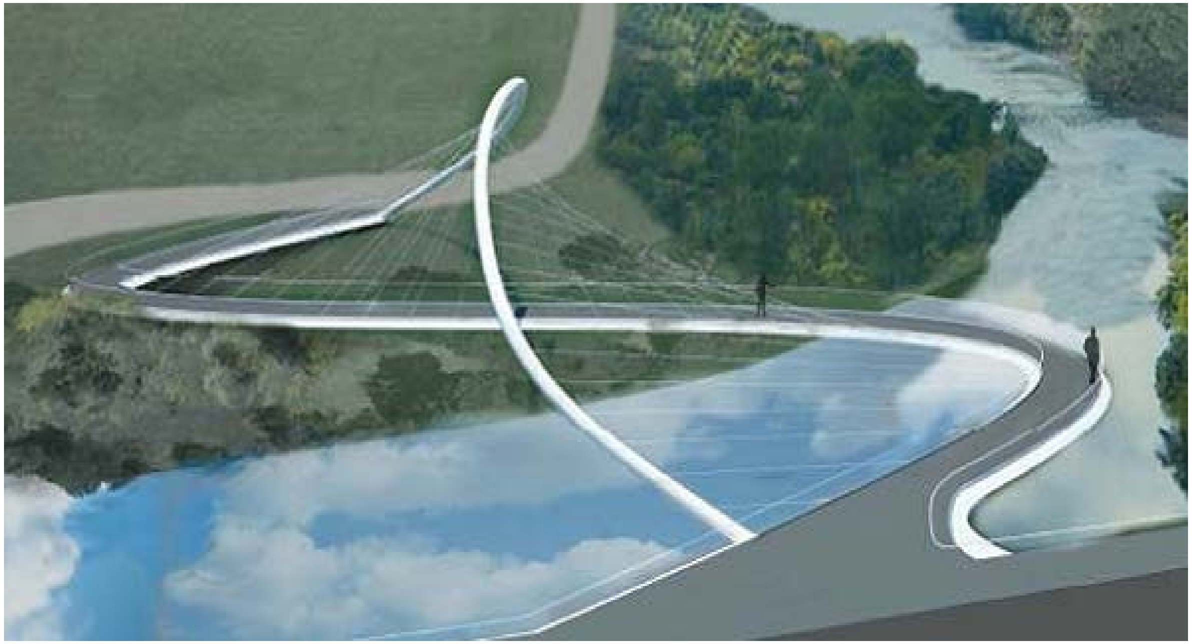
Sketch of the Golden Horn Bridge designed by Leonardo da Vinci in 1502 (Biblioteque Institute Paris).

Ziggenbach Bridge (1924), Landquart Bridge (1930) and Schwandbach Bridge (1933) by Maillart

In order to be structurally efficient, the structure has to be funicular for its loads.

- **First solution: to adapt the geometry of the structure**

Mettingen footbridge proposal by SBP
(Stein 2010)



Salford footbridge proposal by Romo
(Romo 2014)


Ripshorst Footbridge by SBP (SBP 2014)

The geometry of the Ripshorst Footbridge arch was obtained with a hanging model (Schober 2003)

Research Question:

TO DEVELOP AN INTEGRATED AND INTERACTIVE APPROACH TO CONCEPTUAL DESIGN OF ARCH BRIDGES WITH CURVED DECK

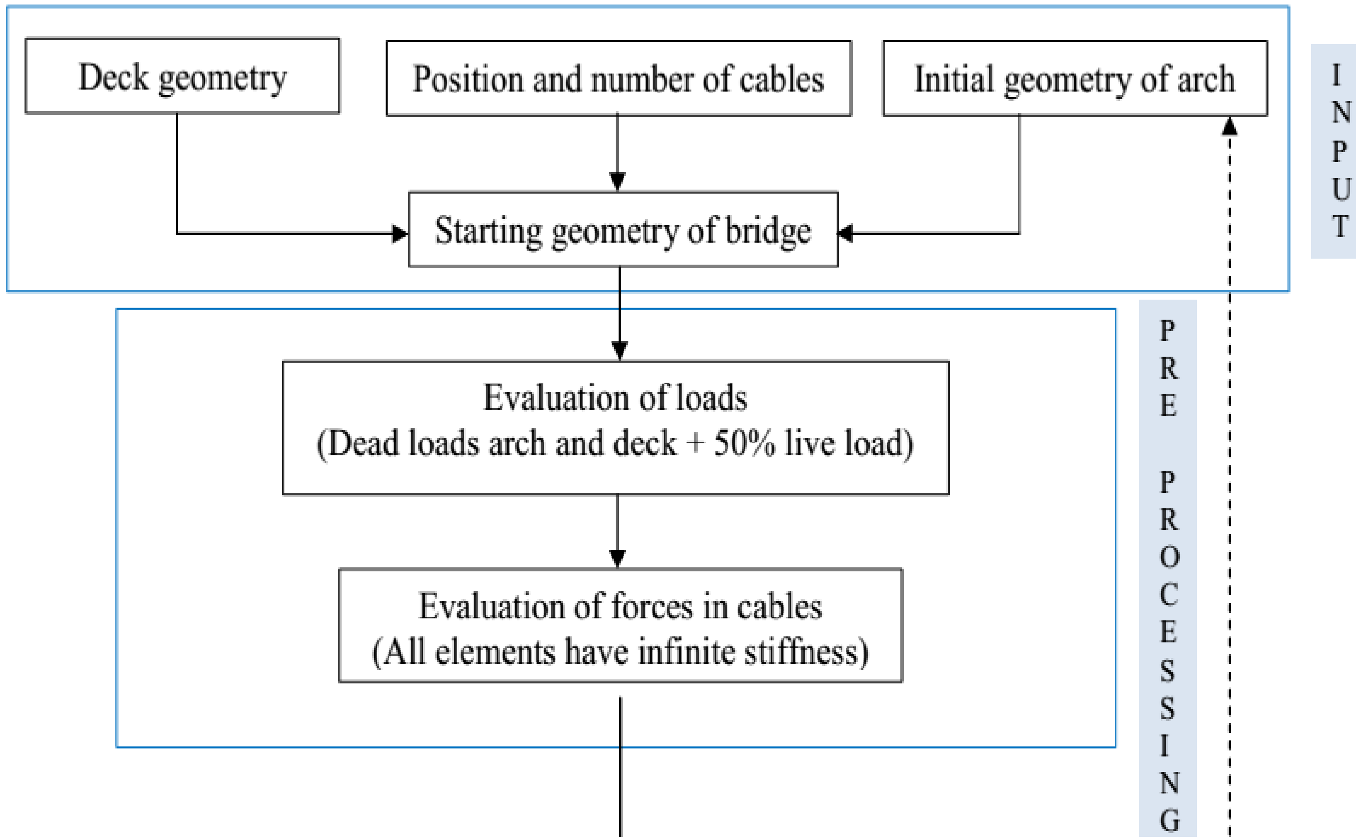
SOFIA (Shaping Optimal Forms with an Interactive Approach) 1/2

The method for finding the antifunicular shapes of an arch consists in an virtual extension of hanging models to 3D.

-the geometry of deck, the number and the position of hangers, and the initial geometry of arch are necessary data for the first model of the bridge. The geometry of the deck is arbitrarily curved;

-loads are evaluated based on the dominant load case. All dead loads, related to arch and deck, and one half of live loads are taken into account;

- forces acting on cables are evaluated taking into account an infinite stiffness of the hangars;



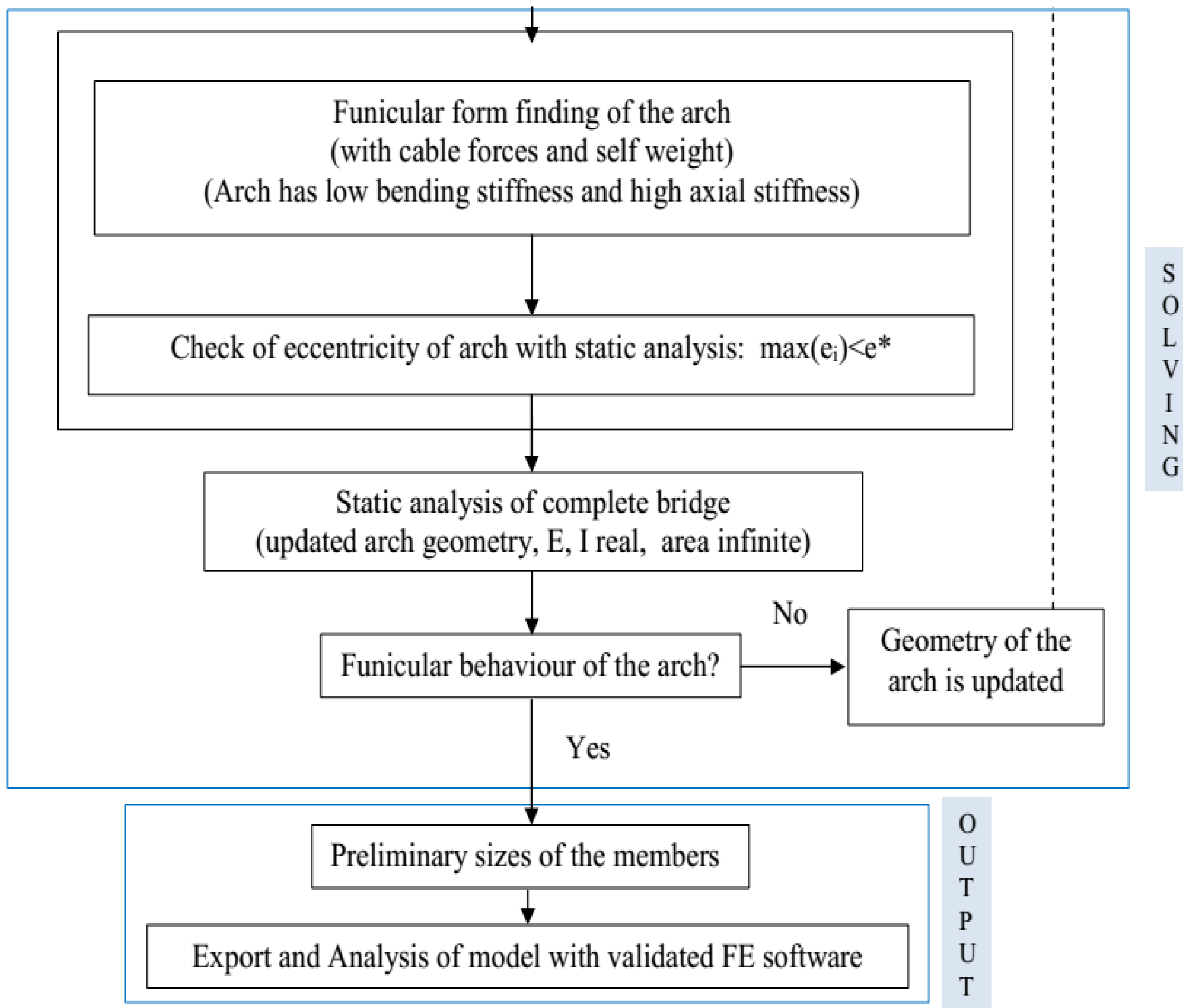
SOFIA (Shaping Optimal Forms with an Interactive Approach) 2/2

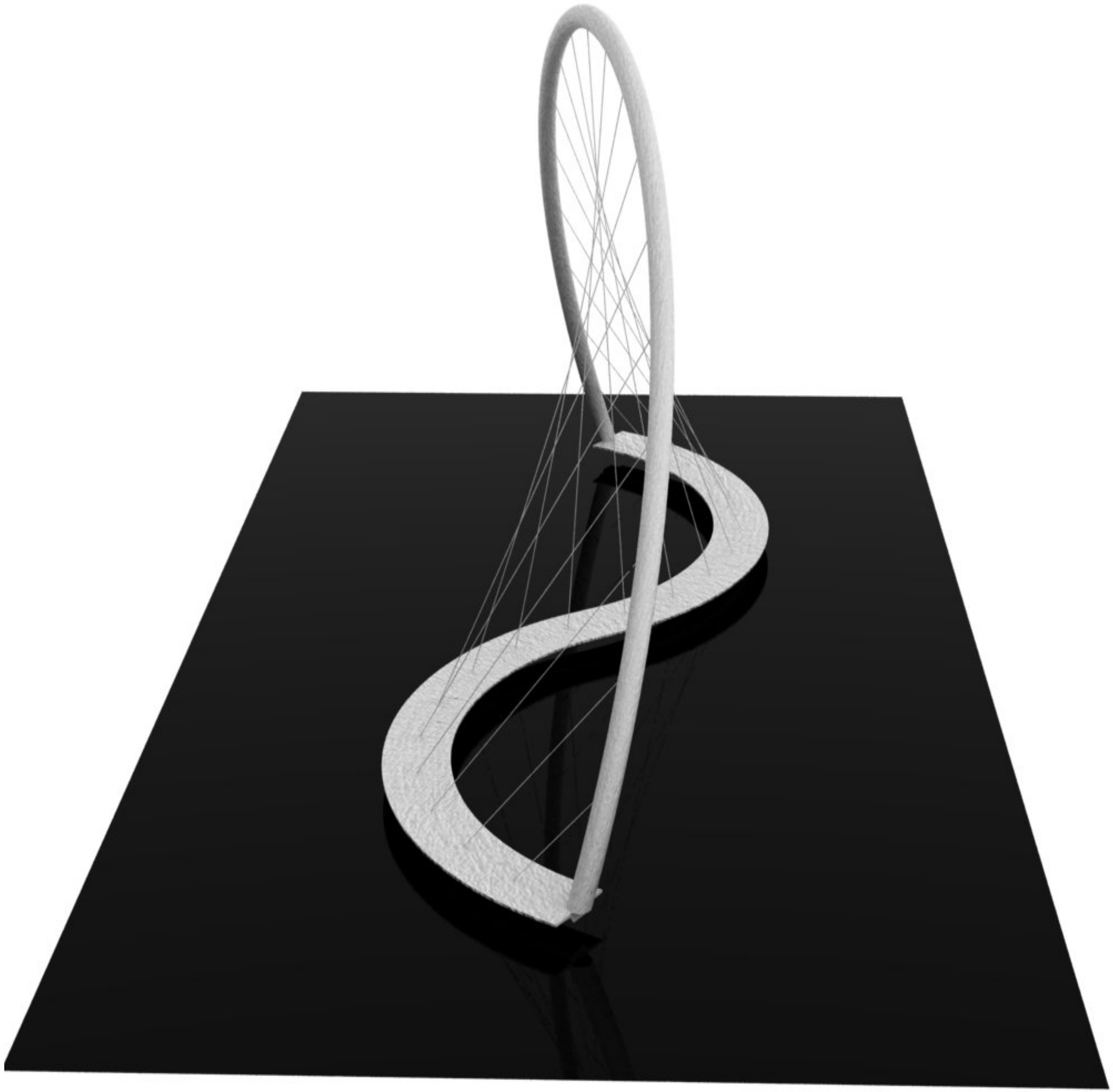
- the form finding analysis of the arch is performed taking into account the axial forces of the hangars and the dead load of the arch. The analysis is stopped when a funicular curve is reached:

$$e_i = \frac{\sqrt{M_i^{y^2} + M_i^{z^2}}}{N_i} < e^*$$

-the bridge is regenerated with the deck and the updated geometry of the arch. The funicular behaviour of the arch is checked. If bending moments are too big, the updated geometry of the arch is used for a new iteration. If the funicular configuration has been reached the form finding analysis is completed;

- once final geometry of the arch is found, preliminary sizes of the members can be determined using the computed forces.

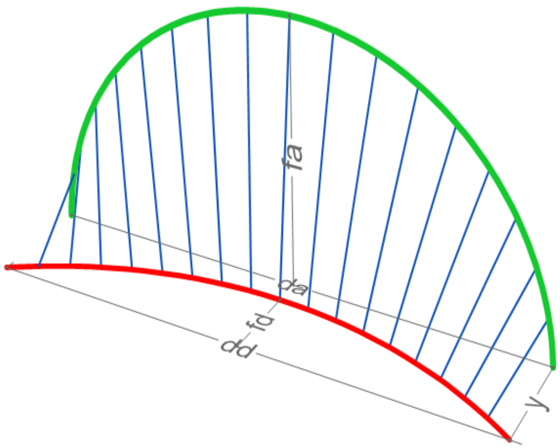




antifunicular arch for bridges with S-shaped deck

antifunicular arch for bridges with C-shaped deck

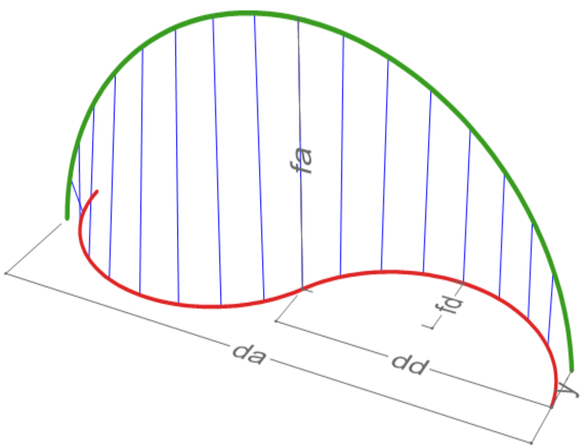
The optimal deck-arch relative transversal position has been investigated for obtaining the most cost-effective bridge.



- The bridge is only subjected to vertical loads.

- Dead load of the arch corresponds to an arch with a steel hollow circular transversal section with diameter of 2m and thickness of 17mm.

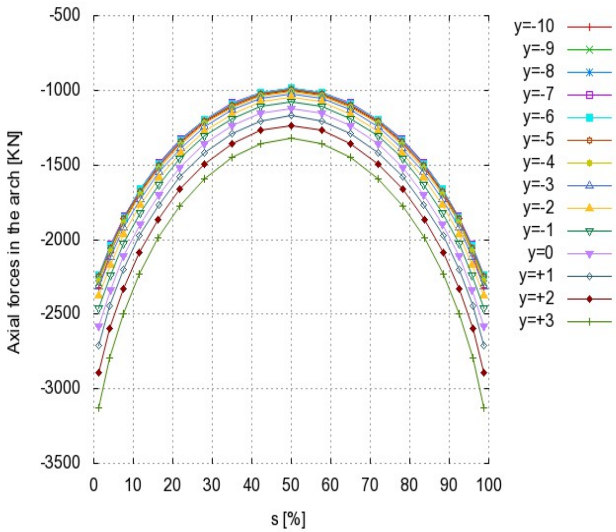
- The load of the deck corresponds to a steel deck with a width of 5m, 3,50 kN/m² of dead load and one half of live load, 2,50 kN/m².



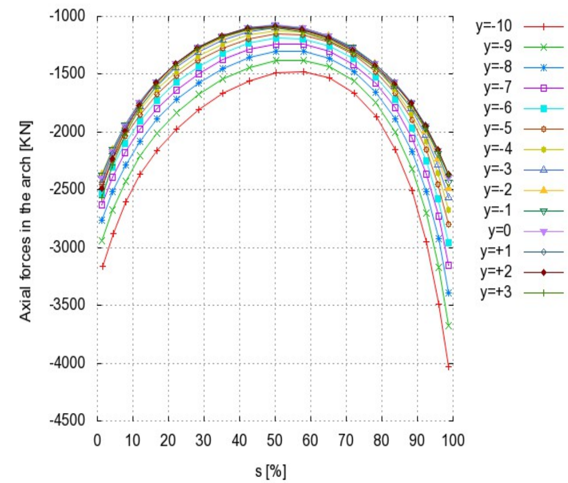
[m]	C-Shape	S-Shape
d_a	100	100
f_a	45	45
d_d	100	50
f_d	10	10
y	$-10<y<3$	$-10<y<3$

Millennium Bridge in Gateshead (Davey and Forster 2007)

Axial forces along the arch vs. curvilinear coordinate of the arch, for different values of y

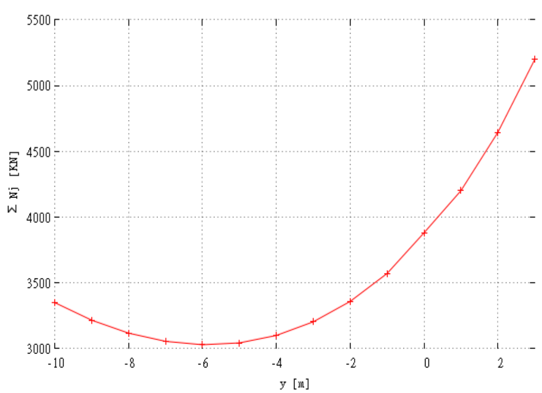


C-shaped deck

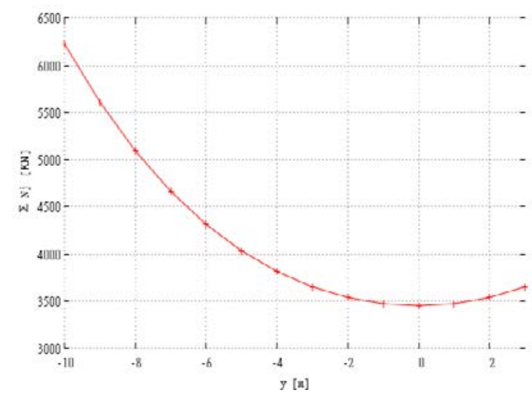


S-shaped deck

Summation of axial forces of all j cables vs. different values of y



C-shaped deck



S-shaped deck

The best position of the deck for obtaining the most cost-effective bridge corresponds to the situation for which the distance between deck's center of gravity and vertical plane passing from arch's abutments is minimum.

The concept of funicularity

Computational Tools for conceptual design

Arch bridges with curved deck

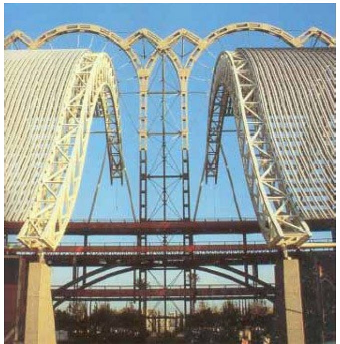
Funicular post-tensioned structures

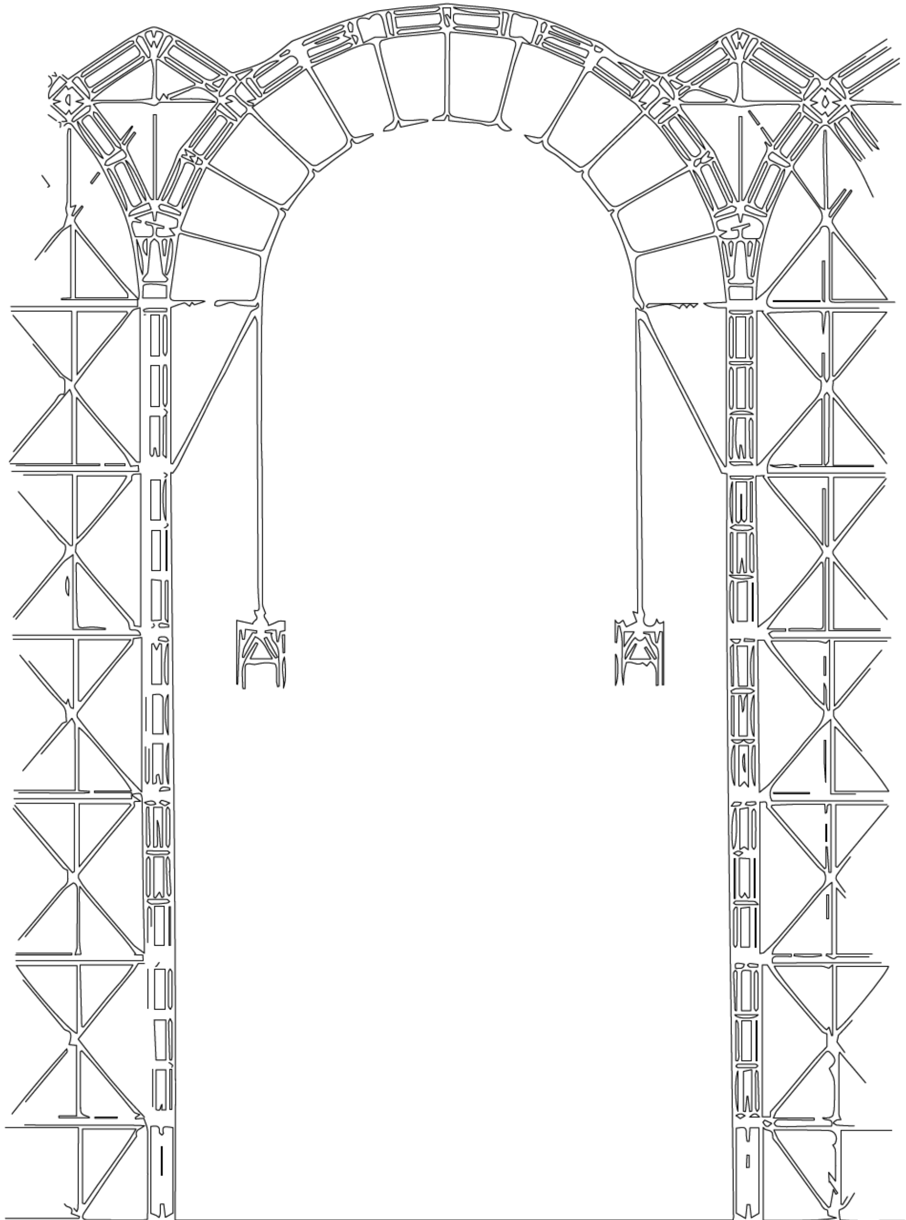
Conclusions and future researches

In order to be structurally efficient, the structure has to be funicular for its loads.

- First solution: to adapt the geometry of the structure

- Second solution: to change the load distribution



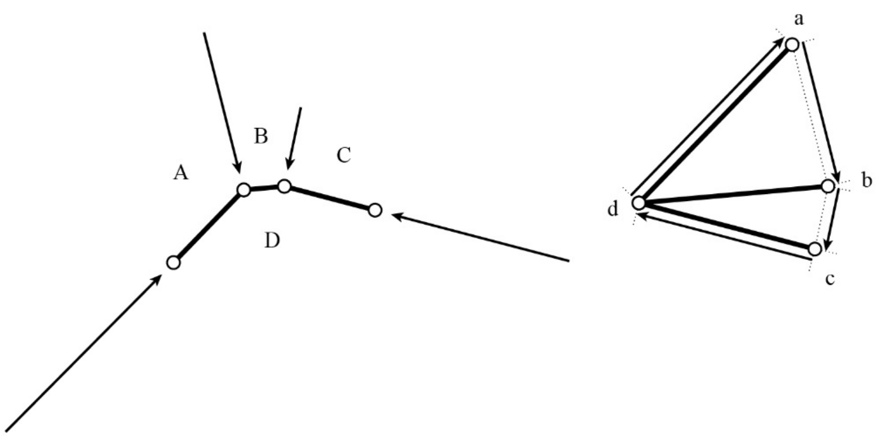

Expo Seville 1992, Peter Rice (ARUP)

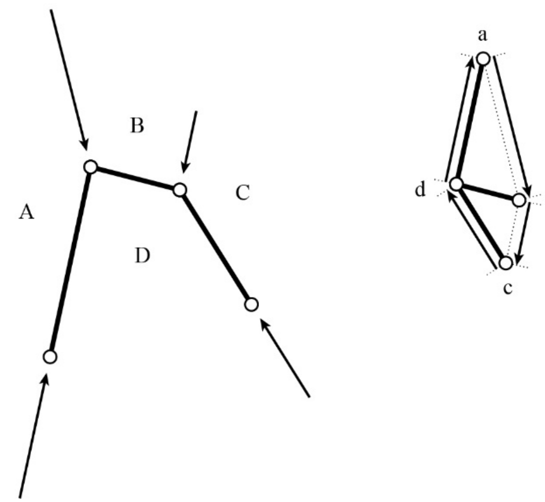
Can external post-tensioning systems convert arbitrary geometries into funicular structures?

Yes. It is possible to find a solution of the problem using Graphic Statics.

There is a need for a generalized theoretical framework and a design-oriented tool that shows how this concept can be applied to designs in a broad sense.

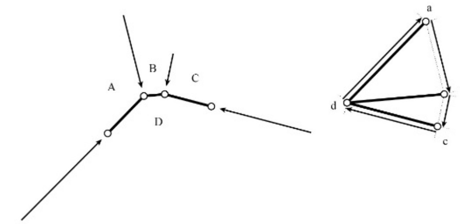
Simplified graphic statics construction 1/2

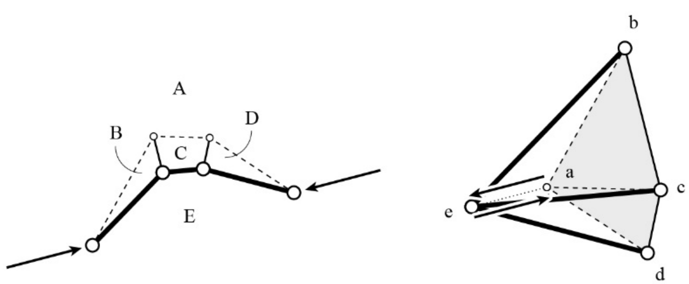


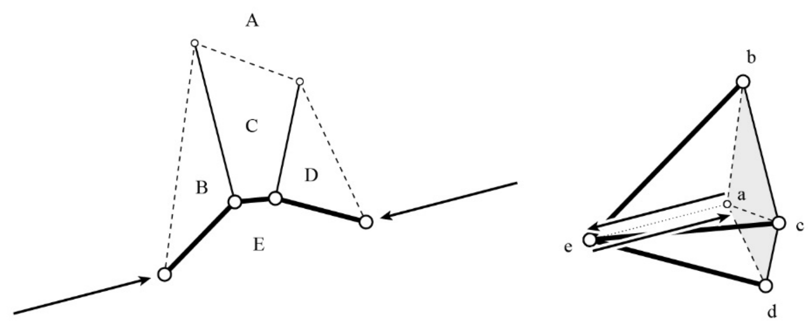


I indeterminacy

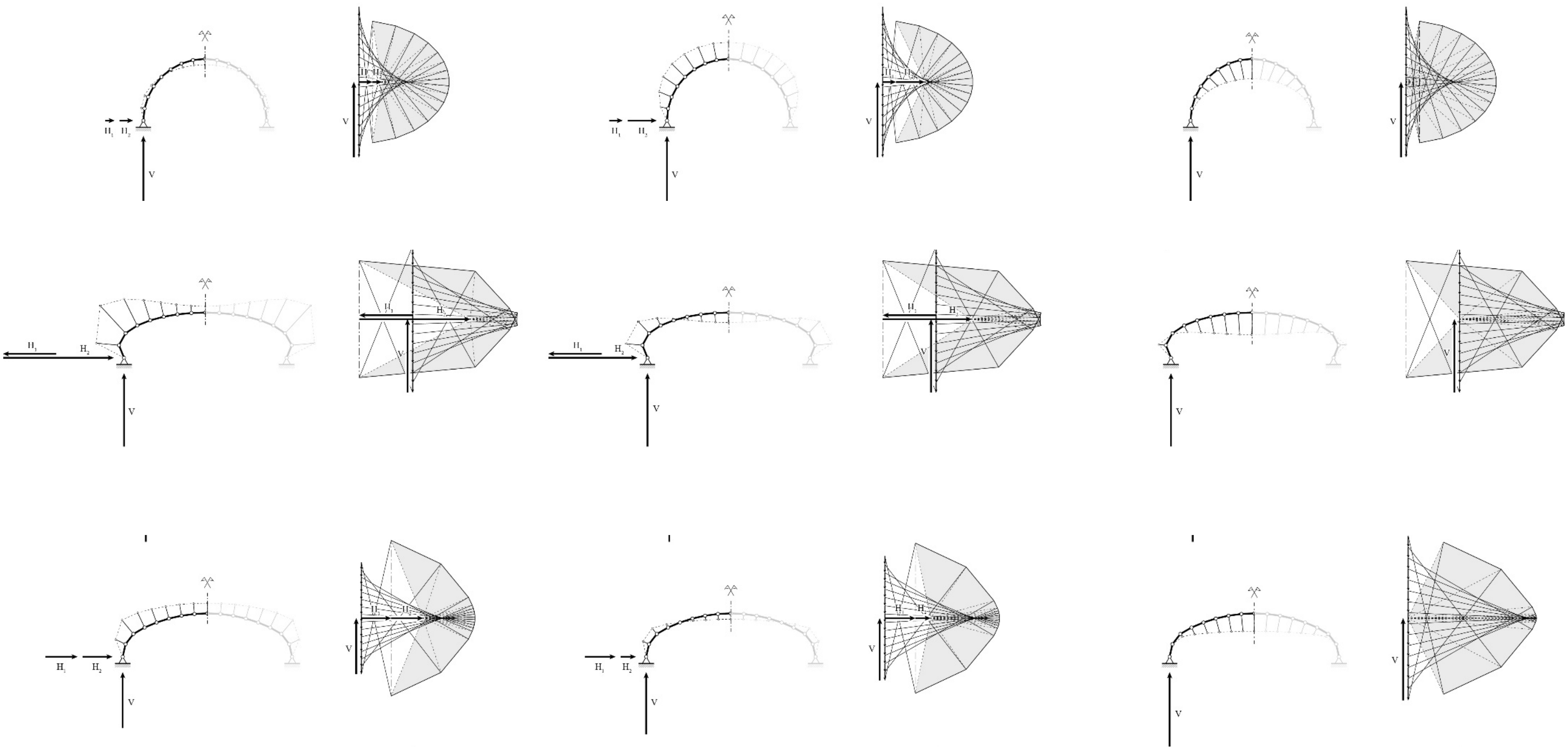
Simplified graphic statics construction 2/2





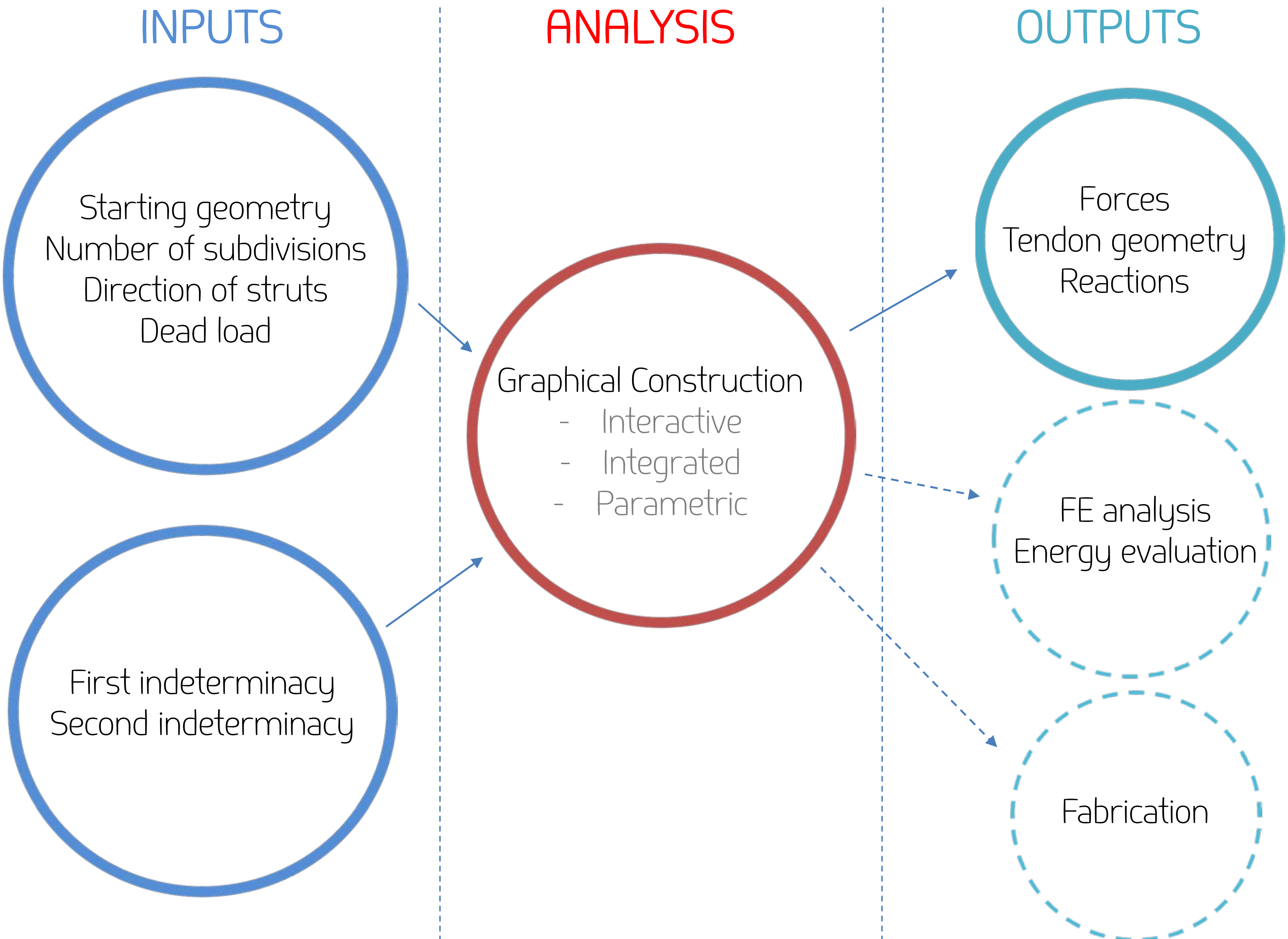


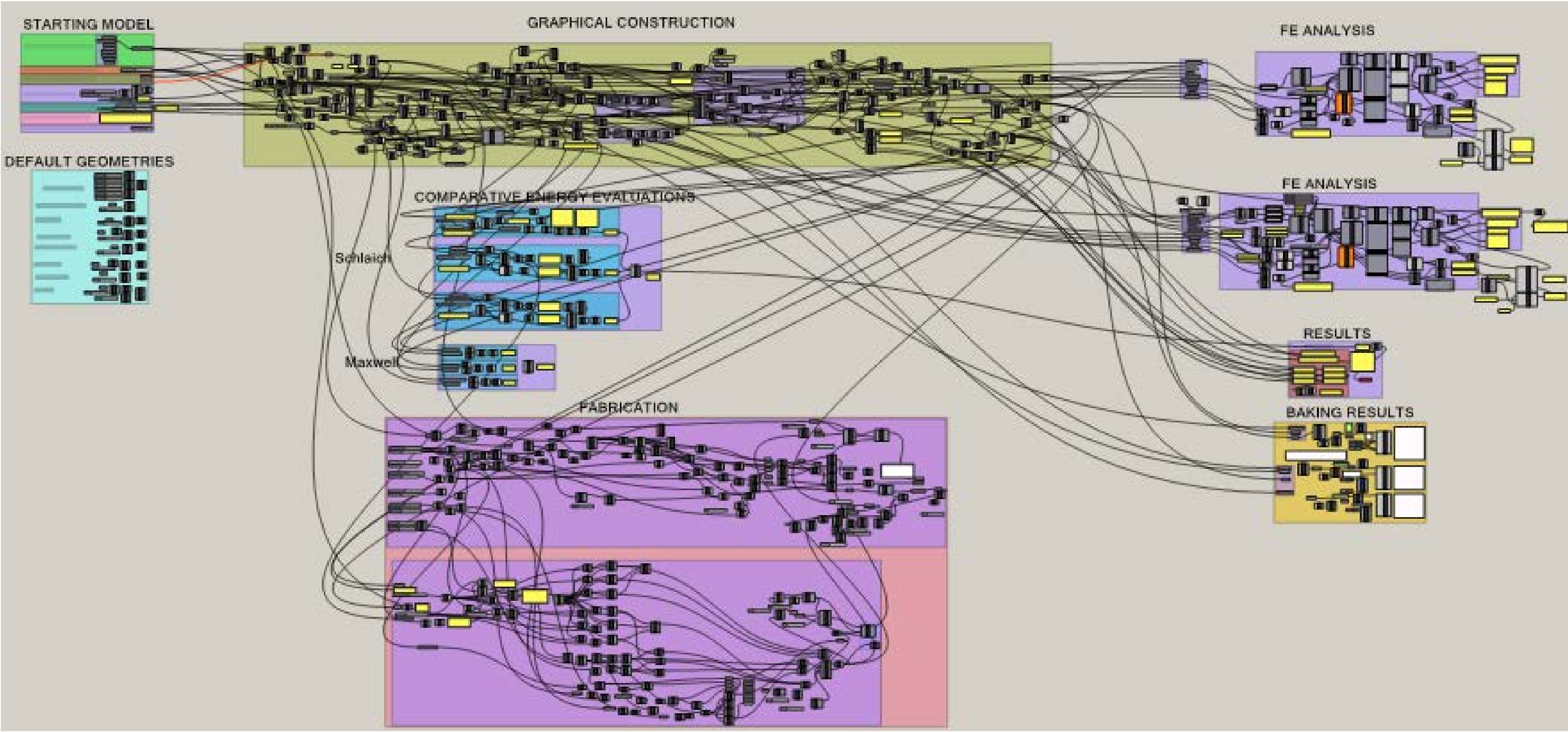
II indeterminacy



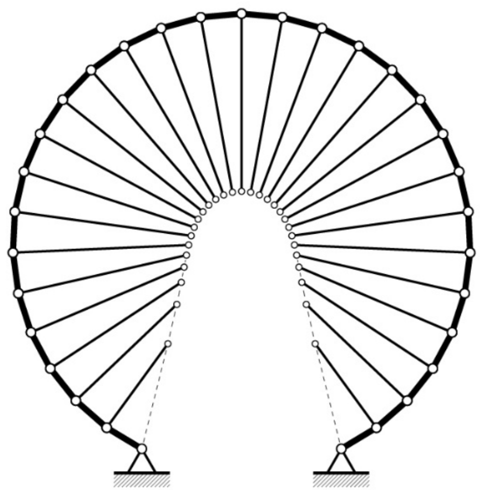
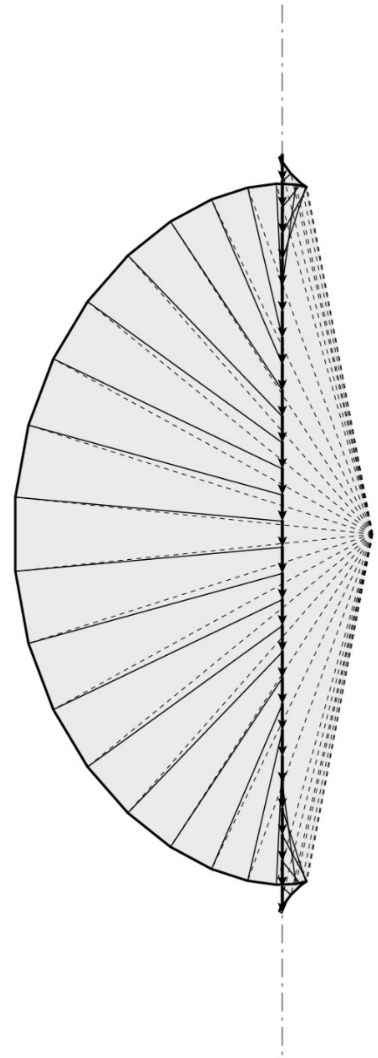
H_1
H_2
V

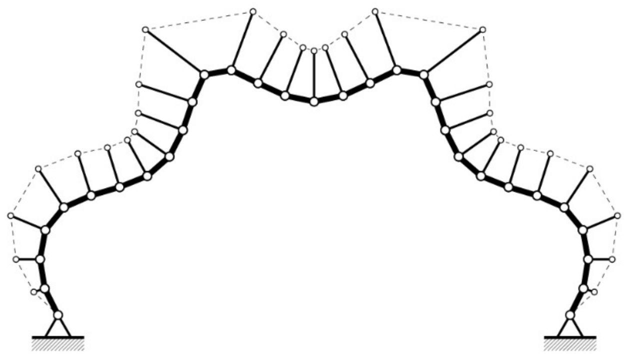
Workflow in Grasshopper

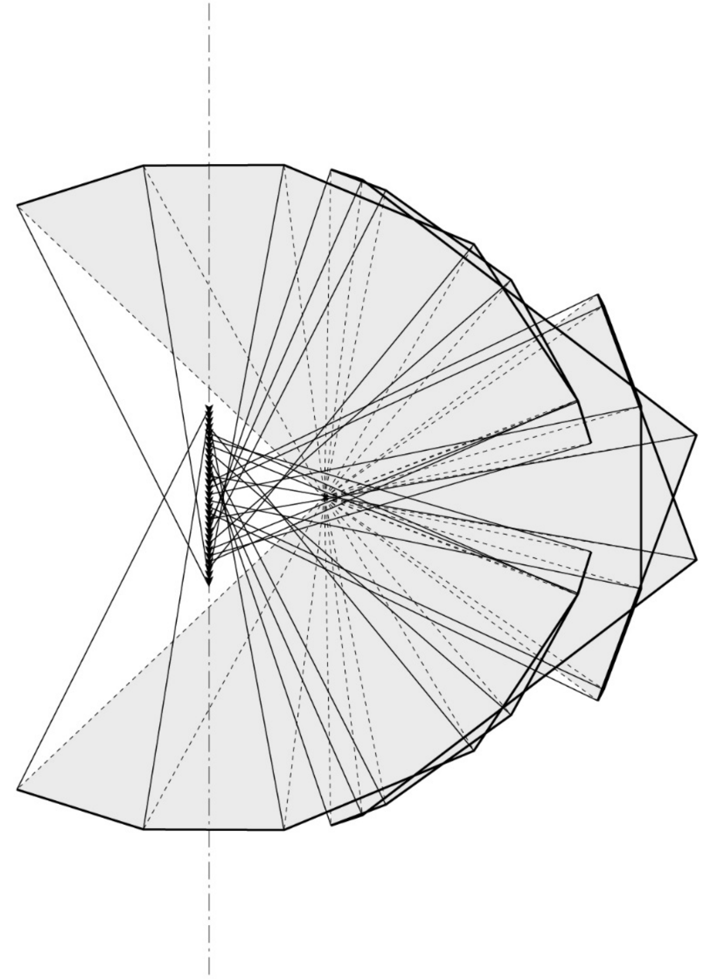


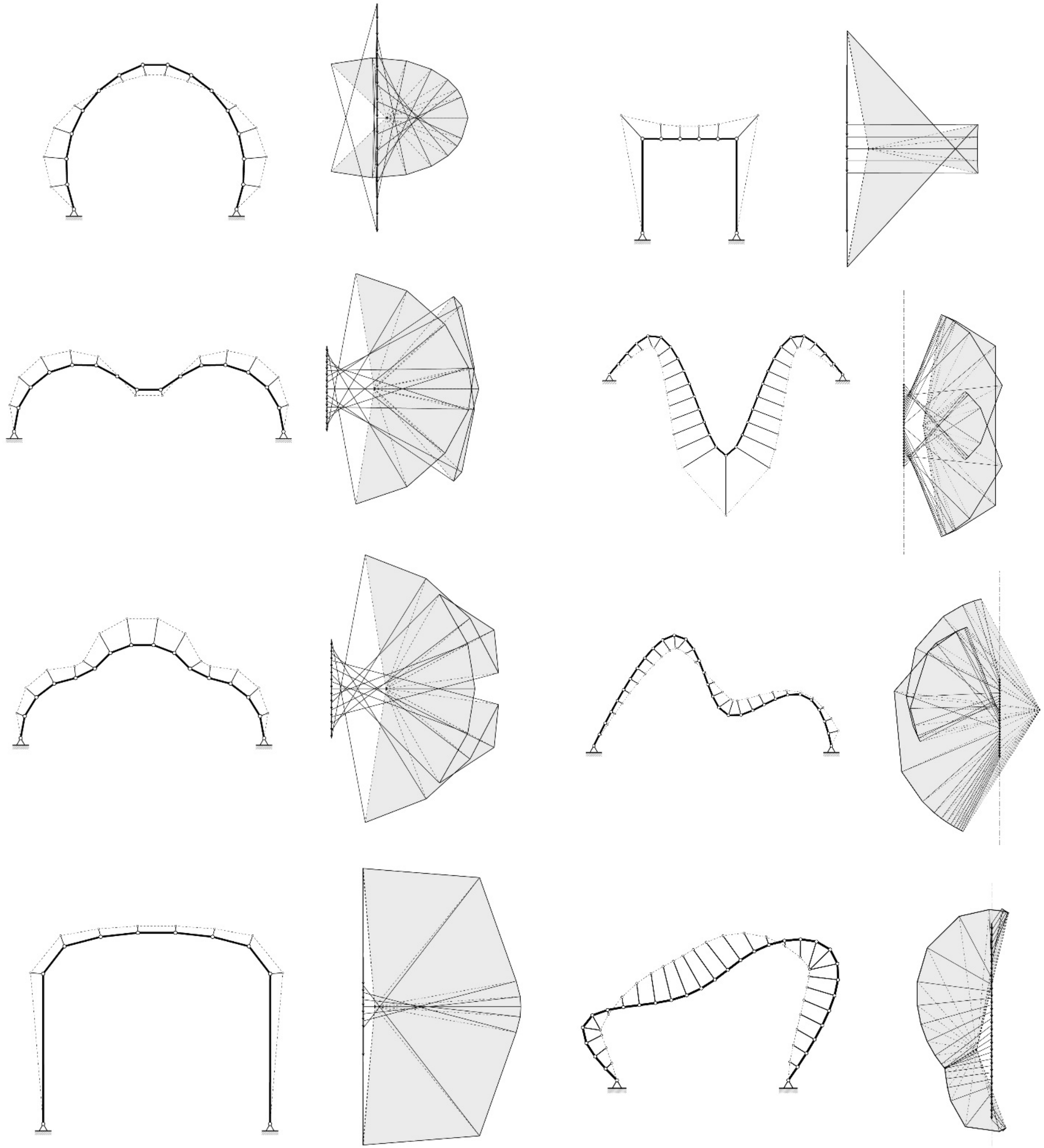


The tool will be made user friendly and free downloadable in the future

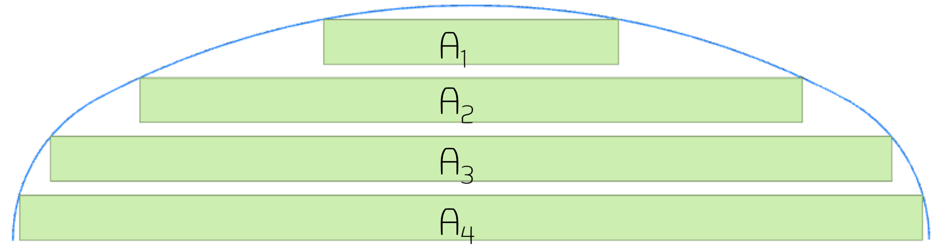






Data

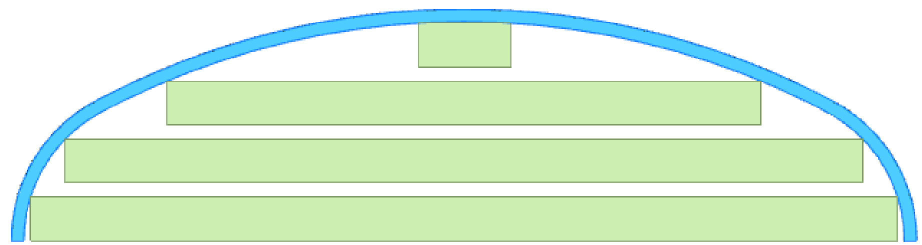
Span=50m
Rise=12m
Roof Load=2kN/m^2
Arches distance=10m
q=20kN/m
HEA Profile – S235



$A^* = A_1 + A_2 + A_3 + A_4$

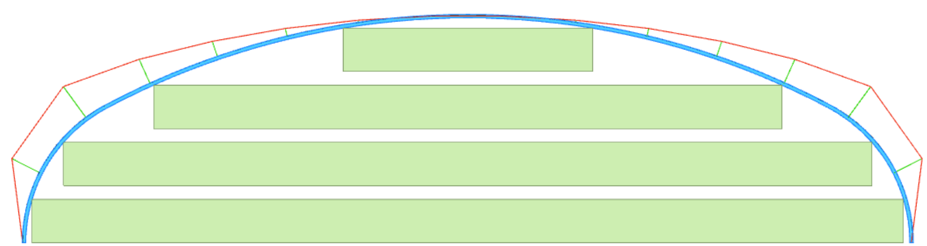
Bending solution

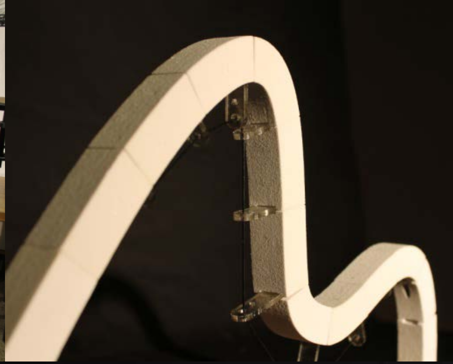
N_{max}=374kN
e=max(Mi/Ni)=2.9m
t=700mm
M*=6704kg

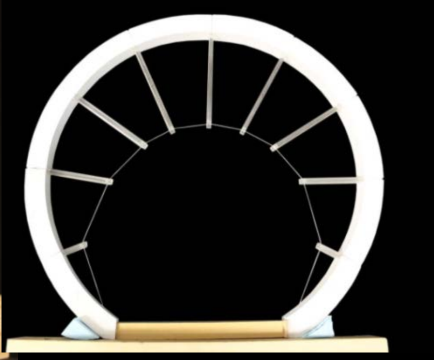


Active solution

N_{max}=660kN
e=max(Mi/Ni)=0.0m
t=180mm
M=37%M*






Photron
FASTCAM SA5 mod... 1000 fps
1/2000 sec
1024 x 1024
End
frame : -2682
-00:00:02.682000000
Date : 2015/1/23
Time : 15:04

The concept of funicularity

Computational Tools for conceptual design

Arch bridges with curved deck

Funicular post-tensioned structures

Conclusions and future researches

Conclusions

- the concept of funicularity is simple and powerful.

- there is a strong need of design tools that have to be interactive and integrate.

- the combination of strong theoretical knowledge and innovative computational tools allows the design of new high-performance structural tipologies

Future researches

- Can fabrication aspects be introduced as parameters in the form finding process?
- Can we use CNC machines and CAD-CAE-CAM tools to create cheap scaffolding with any geometry ?
- Can we explore the creative process of one designer through his original drawings?
- Can we easily develop active structures with Arduino and Grasshopper?
- Can we evaluate structural and environmental performances with the same tool?

Thanks for your attention.

Leonardo Todisco
lt@he-upm.com