On Structural Design as Research Topic

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Thanks to Cervera, Estevan, Hernando, Ortiz, Vázquez, . . .

At first glance, just with several good analysis methods and an exhaustive search of the candidate solutions space with Gödel's numbering, an optimal solution can be found for any structural problem.

But the computational cost of this "design method" would be intractable in a formal sense (NP-completeness theory).

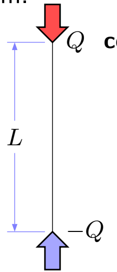
Furthermore, it can be the case that there exists no solution.

So, it is worth of consideration a design theory as a different one that an analysis theory.

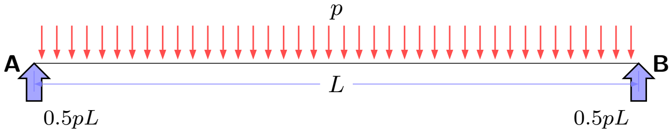
1. **Analysis & Design**
2. **The design theory: two examples**
3. **The design theory: a short tour**
4. **The design theory: issues "to do"**

Analysis *versus* Design: shared elements

The structural problems. A fairly large subset of them can be defined as a set of known forces in static equilibrium (the Maxwell's class). Let be $\mathcal{P}$ one of them.



compression—column, tower



bending—beam, bridge

The structural requirements, $\mathcal{R}$. Strength, stiffness, stability,... Here, only strength will be considered in the classical fashion: $\sigma \leq \mathbf{f}$

The structural solutions, $\{\mathcal{S}_1, \ldots\}$. A set of bodies with suitable shapes for the problem, of any material with known physical properties. Here, only allowable stress, $\mathbf{f}$, and weight density, $\boldsymbol{\rho}$, will be considered.

Analysis *versus* Design: different questions

Let be $\mathcal{P}$ a definite structural problem subject to $\mathcal{R}$.

Analysis Let be $\mathcal{S}$ a (guessed) solution.	**Design** Let be $\mathcal{G}$ a set of additional requirements over the solutions of $\mathcal{P}$.
Are the structural requirements $\mathcal{R}$ satisfied by $\mathcal{S}$ for $\mathcal{P}$?	What is the subset of feasible solutions for $\mathcal{P}$ for which $\mathcal{R}$ and $\mathcal{G}$ are fulfilled?
How far is $\mathcal{S}$ from exactly satisfying any condition in $\mathcal{R}$?	What is the subset of feasible solutions for $\mathcal{P}$, $\mathcal{R}$ and $\mathcal{G}$ for which one of the requirements is exactly satisfied?
How is the performance of $\mathcal{S}$ with regard to any magnitude of interest?	What is the best solution of the feasible set respect to any cost of interests? i.e., what is the solution of minimal cost?

Analysis *versus* Design: different approaches

Analysis: Given $\mathcal{P}$ and $\mathcal{S}$, calculate $\mathcal{R}$

Design: Given $\mathcal{P}$ and $\mathcal{R}$ (and probably $\mathcal{G}$), calculate $\mathcal{S}$

No surprising, as we have an analysis theory, we have a design theory too. The latter came first (GALILEO) that the former (if we put aside the works of LEONARDO).

Remarks: $\mathcal{G}$ stands for no-structural requirements. Some of them can be computable, but someothers aren't.

$\mathcal{G}$ is included into the guessed solution $\mathcal{S}$ in the analysis case.

Analysis *equal* Optimum Design: an intersection point

Let us consider an abstract, structural optimization problem subject to equilibrium constraints:

$$\mathsf{opt}\,\|\boldsymbol{q}\| \quad \text{with} \quad \boldsymbol{Q} = \boldsymbol{H}\boldsymbol{q}$$

The Lagrange's formula will give:

$$\frac{\partial}{\partial \boldsymbol{q}}\Big(\|\boldsymbol{q}\| + \boldsymbol{\lambda}^T(\boldsymbol{Q} - \boldsymbol{H}\boldsymbol{q})\Big) = \boldsymbol{0} \quad \Rightarrow \quad \boldsymbol{H}^T\boldsymbol{\lambda} = \frac{\partial\|\boldsymbol{q}\|}{\partial \boldsymbol{q}}$$

Let us consider two simple examples of $\boldsymbol{q}$ metric:

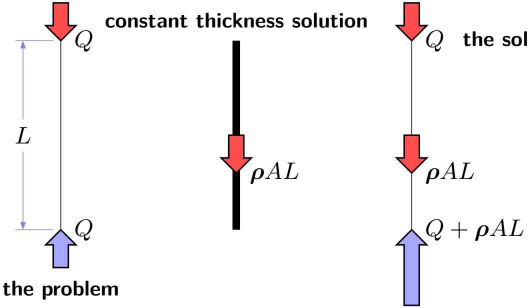
$\|\boldsymbol{q}\| = \frac{1}{2}\boldsymbol{e}^T\boldsymbol{q}$ with $e = q \div k$
$\boldsymbol{\lambda} = \boldsymbol{u}$ and $\boldsymbol{e} = \boldsymbol{H}^T\boldsymbol{u}$

Linear analysis as usual

$\|\boldsymbol{q}\| = L_i \cdot \mathsf{abs}\,(q_i)$
$\boldsymbol{\lambda} = \boldsymbol{u}^\star$, $\boldsymbol{e}^\star = \boldsymbol{H}^T\boldsymbol{u}^\star$
and $e^\star = L \cdot \mathsf{sgn}(q)$
Volume optimization or...
¡plastic analysis!

See Rozvany' or Prager' papers for additional formulations. Nevertheless, a design theory **is not the same that** a structural optimization theory, so the previous differences remain.

The design theory: the minimal problem



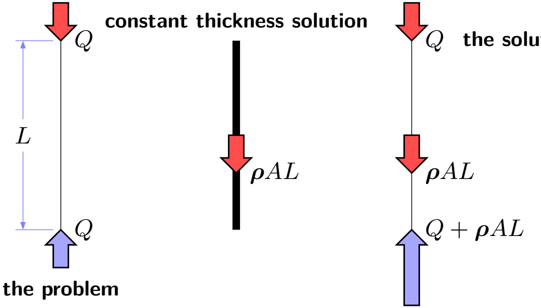
Analysis: $\sigma = \dfrac{Q+\boldsymbol{\rho} AL}{A} \leq \mathbf{f}$?

Some conclusions for designing:

$$\min A = \frac{Q}{\mathbf{f}-\boldsymbol{\rho} L} \qquad \min \text{self-weight} = Q\frac{\boldsymbol{\rho} L}{\mathbf{f}-\boldsymbol{\rho} L} = Q\frac{L}{\frac{\mathbf{f}}{\boldsymbol{\rho}}-L}$$

$\mathbf{f}/\boldsymbol{\rho}$ is a characteristic length of the material, its **structural scope**, $\mathcal{A}$. In this case, it is also the scope of the constant thickness solution (as structural layout), $\mathcal{L}=\mathcal{A}$, but generally $\mathcal{L}=f(\mathcal{A},\ldots)$.

The design theory: the minimal problem



$$\boxed{\textbf{Analysis: } \sigma = \frac{Q + \boldsymbol{\rho} AL}{A} \leq \mathsf{f}?}$$

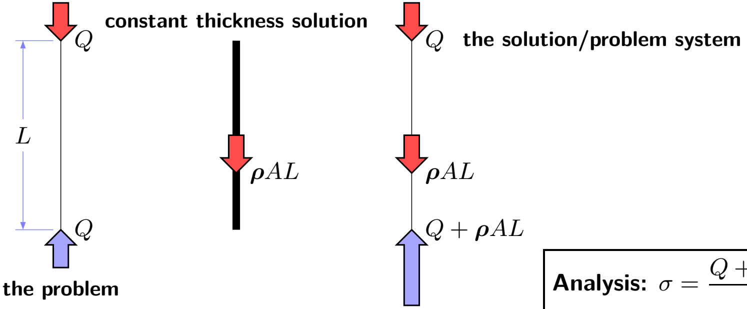
Some conclusions for designing:

$$\min A = \frac{Q}{\mathsf{f} - \boldsymbol{\rho} L} \qquad \min \text{self-weight} = Q\frac{\boldsymbol{\rho} L}{\mathsf{f} - \boldsymbol{\rho} L} = Q\frac{L}{\frac{\mathsf{f}}{\boldsymbol{\rho}} - L}$$

With this simple view, we can write:

$$\text{efficiency: } r = \frac{\text{net load}}{\text{total load}} = \frac{Q}{Q + \boldsymbol{\rho} AL} = 1 - \frac{L}{\mathcal{L}} \qquad \text{load cost: } C = \frac{1}{\text{efficiency}}$$

The design theory: the minimal problem



$$\boxed{\textbf{Analysis: } \sigma = \frac{Q + \boldsymbol{\rho} AL}{A} \leq \mathbf{f}?}$$

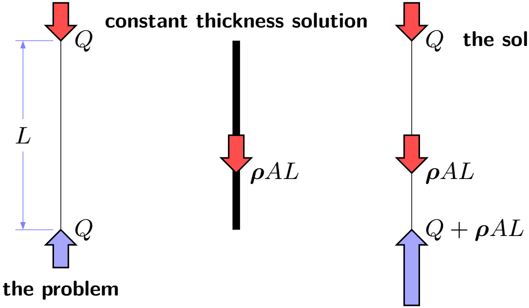
Some conclusions for designing:

What happens if we would know in advance the scope, $\mathcal{L}$, of a set of similar solutions for a stated Maxwell's class of problems of size L?

We would know in advance:

- the efficiency of the solution, $r \leq 1 - L/\mathcal{L}$;
- its material volume as a fraction of the net or useful load, $\geq \dfrac{1-r}{r} \times \dfrac{Q}{\boldsymbol{\rho}}$;
- and the remaining tasks will be defining its geometry and details.

The design theory: the minimal problem



Analysis: $\sigma = \dfrac{Q + \boldsymbol{\rho} AL}{A} \leq \mathbf{f}$?

Some conclusions for designing:

What happens if we would know in advance the scope, $\mathcal{L}$, of a set of similar solutions for a stated Maxwell's class of problems of size L?

Note that we also know in advance if a stated problem is unsolvable.

If we know the scope for the best layout, we know that all the problems with $L > \mathcal{L}$ have no solution. (If we do not know if the layout is actually the best, we know that these problems have no solution with this layout: we must look for a better one!)

The design theory: an everyday example

Sizing the cross-section A with shape $\mathcal{S}$ of a member of length L supporting axial force q

Rule for...	Analysis	Design
Tension	$\sigma_c = \dfrac{q}{A} \leq \mathbf{f}$	$A \geq \dfrac{q}{\mathbf{f}}$
Compression	$\sigma_c = \dfrac{q \cdot \omega(\lambda, \mathcal{S})}{A} \leq \mathbf{f}$ with $\lambda = \lambda(L, i)$, $i = i(A, \mathcal{S})$	To solve for A: $A\mathbf{f} = q \cdot \omega(\lambda, \mathcal{S})$ **This is not a rule!**

The analysis rule cannot be easily transformed in design rule because of the algebraic complexity of ω.

The design theory: an everyday example

Destiling a approximate design rule for the compression case:

a Aesthetic requirement: for a good design it should be $\omega \leq 2$ (ω is a factor of efficiency of the layout).

b The ratio L^2/q is invariant for solutions with **similar shape** but **different size**, $\dfrac{L^2}{q} = \dfrac{A\mathbf{f}}{\lambda^2 \cdot \omega(\lambda) \cdot \mathcal{E}^2}$, where $\mathcal{E}$ is a property for each cross-section **shape**. We can write several proportions between invariants, for example: $\lambda/\lambda' = L/L'$.

c We write $\frac{A\mathbf{f}}{\omega} = q \quad \rightarrow \quad \frac{A\mathbf{f}}{\omega} + A\mathbf{f} = q + A\mathbf{f} \quad \rightarrow \quad A\mathbf{f} = q + A\mathbf{f}\left(1 - \frac{1}{\omega}\right)$: that is, we descompose $A\mathbf{f}$ in two parts: one required by q, the other by stability and **doesn't** depend on q.

d As $\omega(\lambda)$ is another invariant, we search for a good approximation within aesthetic space, and we find that $\left(1 - \frac{1}{\omega}\right) \approx \frac{1}{2}\left(\frac{\lambda}{\lambda_{\omega=2}}\right)^2$

Mixing all these results, we get $A\mathbf{f} \approx q + \alpha \cdot L^2$ with $\alpha(\mathcal{S}) = \left.\dfrac{\mathbf{f}}{\lambda^2 \cdot \omega \cdot \mathcal{E}^2}\right|_{\omega=2}$: a rule for estimating A for each cross-section shape, that only depends of the problem data, q and L, and our selection of the shape. For normal steel and good cross-section, like tubes, $\alpha \approx 10\,\mathrm{kN/m^2}$.

The design theory: a short tour (Contemporary jargon, informal definitions)

The basics are well established by GALILEO from proportional rules:

$$(\boldsymbol{\rho} A)L + Q = (\boldsymbol{\rho} A)L'$$

where L is the volume height of structure and L' is its limits for each material and shape. He examines the vertical tension and simple flexion cases (but with only momentum equilibrium in the later).

With modern materials like steel, with scope of several kilometres, the size of actual structures is small, very small: $L \ll \mathcal{L} \approx \mathcal{O}(\mathcal{A})$. As a consequence, the exact Galileo's rule has no precision: the self-weight is negligible in almost all cases. It is not surprising that these issues have received little attention. (Nevertheless, the interest of the subject is undoubted: if we consider other costs, like carbon dioxide emission or embodied energy, the self-cost would be not negligible when compared with other phases of life cycle: maintenance, use, etc.)

The design theory: a short tour (Contemporary jargon, informal definitions)

A few definitions and warnings.

Each set of known forces in global equilibrium defines a **Maxwell problem**. We **must** know (or select) actions and reactions and its relative position.

A Maxwell's structure is any set of internal forces (tension or compression) in self-equilibrium that added to the external forces of a Maxwell's problem satisfies that every subset of forces, internal or external, acting at each point is in equilibrium (local equilibrium). **There is no self-weight here.**

Any open funicular polygon (parabola or catenary) **is and is not** a Maxwell's structure. We **must either** define arbitrarily some reactions **or** close the polygon: arch and tie, or cable and strut, or arch and cable,. . .

The design theory: a short tour (Contemporary jargon, informal definitions)

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Let us define **the quantity of structure** by:

$$\mathcal{V} = \int_V \mathsf{abs}\,(\sigma)\;\mathrm{d}V \;= \sum_i \mathsf{abs}\,(q_i)\,L_i$$

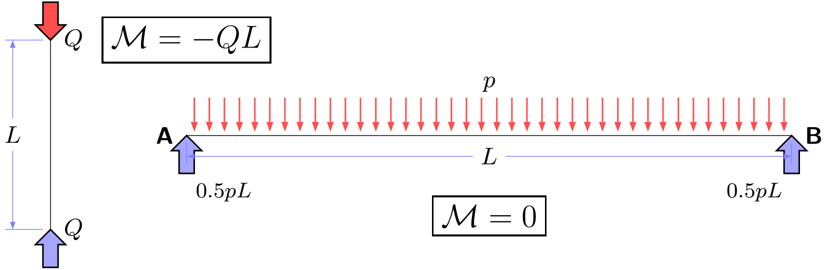
where q is the internal force in each member and L its length; V stands for all the volume of the structure. The definition has sense for any Maxwell's structure.

The design theory: a short tour (Contemporary jargon, informal definitions)

The Maxwell's theorem (ca. 187). For all strut and tie structures that solve a Maxwell's problem the Maxwell's number M is invariant:

$$\mathcal{M} = \int_V \sigma \, \mathrm{d}V = \sum qL$$

(Proof: consider virtual expansion)



The Maxwell's theorem. Corollaries.

- The difference between the quantity of tension structure and that of compression structure is invariant.

  $\mathcal{M} = \mathcal{V}_+ - \mathcal{V}_-$ and $\mathcal{V} = \mathcal{V}_+ + \mathcal{V}_-$ as far as $\mathcal{M} = L_i q_i$ and $\mathcal{V} = L_i \text{abs}\,(q_i)$.

- If any change in the structure definition reduces the quantity in tension (or in compression) it also reduces the other part and the total quantity.

- The quantity of structure determines its minimal volume and weight. When the absolute value of allowable stress is constant, namely **f**, then $V = \mathcal{V}/\mathbf{f}$ and $P = \mathcal{V}/\mathcal{A}$ (**strict sizing**). With self-weight and constant thickness hyphotesis, replace '$=$' with '$>$' because the strict sizing is generally unattainable.

- Any structure only in tension (or in compression) has a minimal quantity of structure and it could be equivalent to any other minimal structure for the same Maxwell's problem (with strict sizing).

  If $\mathcal{V}_- = 0$ then $\mathcal{V} = \mathcal{M} = \mathcal{V}_+$; if $\mathcal{V}_+ = 0$ then $\mathcal{V} = -\mathcal{M} = \mathcal{V}_-$.

Michell's theorem. (1904)

A Maxwell's structure can attain the minimal quantity of structure if the space occupied by it can be virtually deformed, such that the virtual strains in all members of the structure attain the same absolute value and with equal sign than its original stress, and that value is not less than the absolute virtual strain of any line segment of the space.

If the virtually deformed space extents to infinity in all directions, the quantity of structure can be an absolute minimum, otherwise it will be a minimum only relatively to those structures within the same finite space.

The design theory: a short tour (Contemporary jargon, informal definitions)

Michell's theorem. Proof.

Let us consider a Maxwell's problem and all appropriate Maxwell's structures within a given boundary. Now consider that the enclosed space undergoes a virtual deformation such that no linear element has absolute strain greater than ε.

Virtual work principle applied to each structure S gives us:

$$\delta W = \sum_{\mathrm{S}} \Delta \cdot L \cdot q$$

where δW is the virtual work of the known forces, independent of the structure S, and Δ is the virtual strain of each bar.

Then:

$$\delta W = \sum_{\mathrm{S}} \Delta \cdot L \cdot q \leq \sum_{\mathrm{S}} \mathsf{abs}\left(\Delta\right) \, L \, \mathsf{abs}\left(q\right) \leq \varepsilon \sum_{\mathrm{S}} L \, \mathsf{abs}\left(q\right) \leq \varepsilon \mathcal{V}_{\mathrm{S}}$$

The virtual work of known forces is a lower limit of the quantity of structure of any of them.

The design theory: a short tour (Contemporary jargon, informal definitions)

Michell's theorem. Proof.

Let us consider a Maxwell's problem and all appropriate Maxwell's structures within a given boundary. Now consider that the enclosed space undergoes a virtual deformation such that no linear element has absolute strain greater than ε.

$$\delta W = \sum_{\mathrm{S}} \Delta \cdot L \cdot q \leq \sum_{\mathrm{S}} \mathsf{abs}\,(\Delta)\; L\,\mathsf{abs}\,(q) \leq \varepsilon \sum_{\mathrm{S}} L\,\mathsf{abs}\,(q) \leq \varepsilon \mathcal{V}_{\mathrm{S}}$$

If a structure O exists such that $\Delta \cdot q = \varepsilon \cdot \mathsf{abs}\,(q)$ in all parts, the signs of inequality may be replaced by that of equality, and

$$\varepsilon \mathcal{V}_{\mathrm{O}} = \varepsilon \sum_{O} L \cdot \mathsf{abs}\,(q) = \delta W \leq \varepsilon \mathcal{V}_{\mathrm{S}}$$

so that the quantity of structure O is a minimum. Q.E.D.

Remark: In my view, there is no sound proof that a such structure O fulfilling the theorem exists for any Maxwell's problem.

The design theory: a short tour (Contemporary jargon, informal definitions)

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Let us consider a Maxwell's problem and all appropriate Maxwell's structures within a given boundary. Now consider that the enclosed space undergoes a virtual deformation such that no linear element has absolute strain greater than ε.

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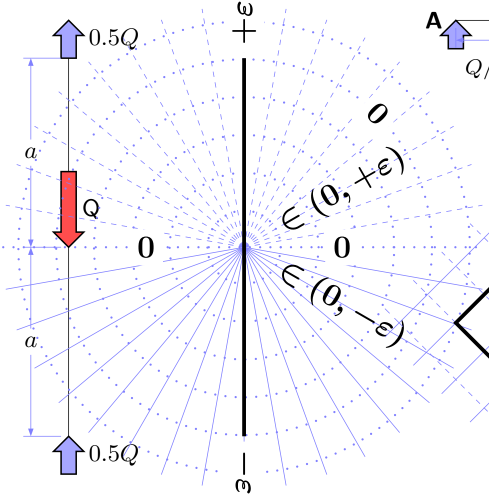
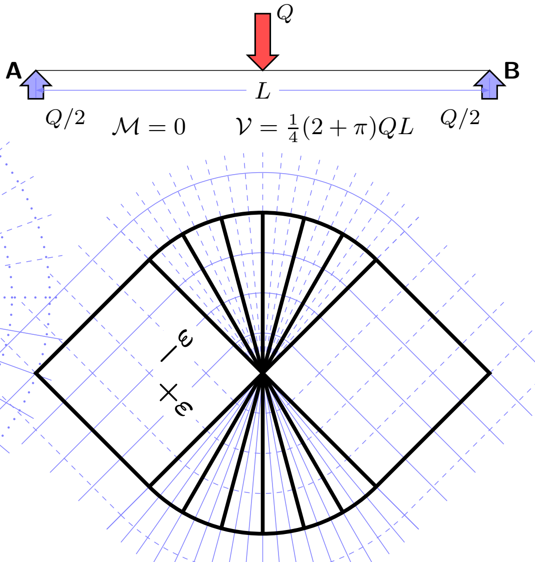
Remark: No hipotesis on constitutive equation is needed if ε is small enough.

Michell's theorem. Examples.



$\mathcal{M} = 0 \quad \mathcal{V} = 2Qa$



The blue lines are the principal directions of strain tensor.

The design theory: a short tour (Contemporary jargon, informal definitions)

Michell's theorem. Optimality criterion.

$$\frac{\partial^2\phi}{\partial\alpha\partial\beta} = 0$$

where ϕ stands for principal direction of virtual strain tensor, and α, β, for orthogonal curvilinear co-ordinates.

The design theory: a short tour (Contemporary jargon, informal definitions)

Michell's theorem. Shape search by graphical methods. The 60's

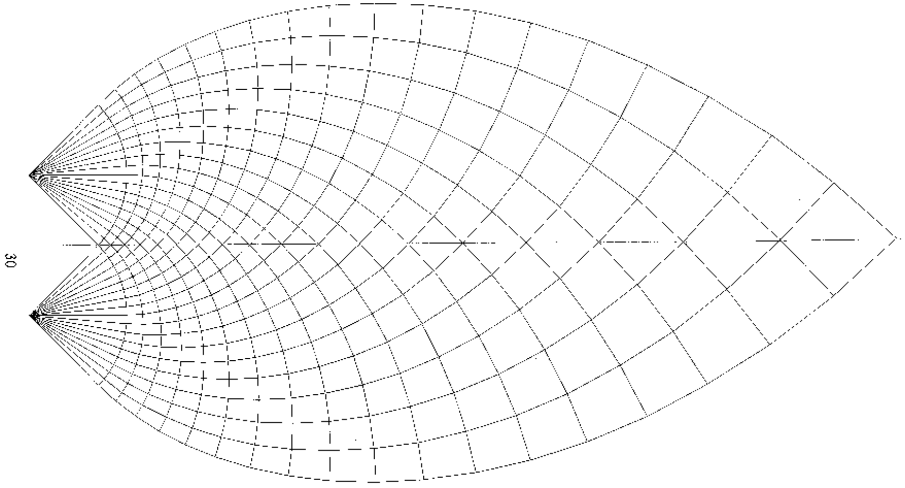
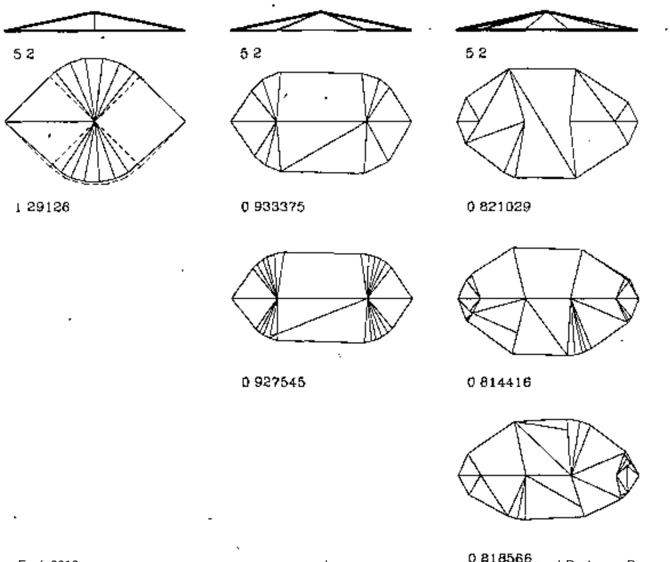


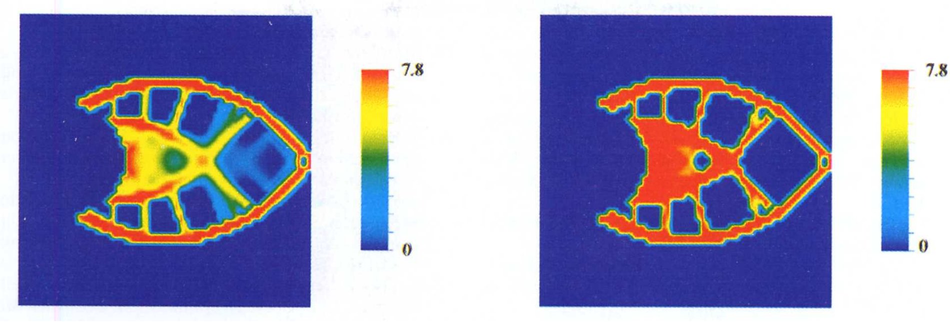
Fig. 16.

The design theory: a short tour (Contemporary jargon, informal definitions)

Michell's theorem. Shape search by simulated annealing. (1995)

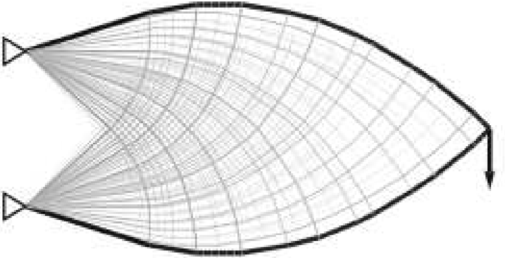


Michell’s theorem. Shape search by Auto-organised chaos (Payten *et alii*, 1997)



Michell's theorem. Shape search by ground structure method.

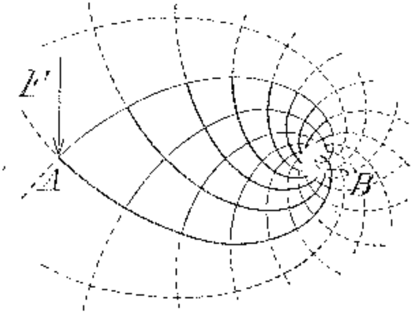
(SOKOL, 2011)



The design theory: a short tour (Contemporary jargon, informal definitions)

The kernel of the theory

A) MICHELL (1904) established an optimality criterion for Maxwell problems in the case $L \to 0$ (or $\boldsymbol{\rho} = 0$). Furthermore, he established the differential equation of the optimal layout. **In the last years, there is a renewed interest on the subject: GS: Michell+truss+FEM+[after 2000] = 235 papers.** (Maybe the centenary of the MICHELL paper?)



The kernel of the theory

B) When the self-weight is isomorphic with the net load (the load of the Maxwell's problem), the rule of Galileo is **exact** among the solutions of similar shape. **This is very useful for quick, every-day designing of common types of structures: constant deep beams and so. . .**

C) Joining the two last points, if we know the Michell solution, we can calculate the maximal scope for a stated problem, and applying the Galileo rule, obtain minimal structural weight and associated costs for each case of the problem with given size. (See the minimal problem above.)

The design theory: a short tour (Contemporary jargon, informal definitions)

The kernel of the theory

B) $$\forall P, Q : \quad \boldsymbol{P} \propto \boldsymbol{Q}$$

strict sizing: $$\forall P, Q : \quad P = \mathcal{V}_{P+Q} \cdot \boldsymbol{\rho} / \mathbf{f} = \mathcal{V}_{P+Q} \div \mathcal{A}$$

A) $$\mathcal{V}_{P=0} = \mu Q L \quad \Rightarrow \quad \mu = \mathcal{V}_{P=0} \div Q L$$

C) $$\mathcal{V}_{Q=0} = \mu P \mathcal{L} = \mu \mathcal{V}_{Q=0} \cdot \mathcal{L} \div \mathcal{A} \quad \Rightarrow \quad \mathcal{L} = \mathcal{A} \div \mu$$

Unformally, the scope $\mathcal{L}$ is an eigenvalue for the relation between P and $P + Q$; so, the role of the scope is analogous than that of the Euler load in stability problem.

The kernel of the theory

D) Last but not least, **the Michell's theorem and its relatives, teaches us very useful rules about the geometric properties of the well conceived structural layouts for cases of small size.**

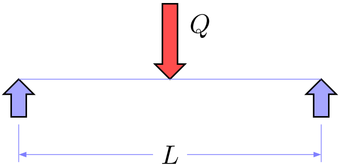
The design theory: a short tour (Contemporary jargon, informal definitions)

Weakness (criticism)

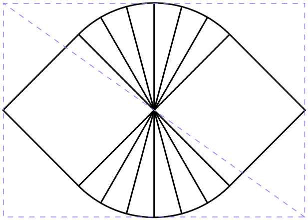
1. "In spite of a prolonged international research effort, Michell layouts have only been determined for a few simple loading conditions" (ROZVANY, 1984). As a consequence, in many problems the estimate for the scope is valid only for some concrete structural layouts, not for the structural problem. ($\min\ \mathcal{V}_{L=0}$ unknown)
2. Generally, the self-weight is not isomorphic with the net load, so that all the theorems over the Maxwell's class of structural problems are at best approximations for real problems, for which the size is not always negligible. ($\boldsymbol{P} \not\propto \boldsymbol{Q}$ and $\mathcal{V} \div QL \neq \mathcal{V}' \div QL'$)
3. Joining the two last points, we can doubt of the existence of a finite limit over the scope for each structural problem or layout. (From my point of view there is no doubt at all: the limit exists although unknown... but my opinion belongs to beliefs, not to facts, as does the opposite one. As far as I know, there is no consistent proof in any of both cases.)

The design theory: a short tour (Contemporary jargon, informal definitions)

Structural Shape (or Form): a design view (no topology here)



A Maxwell's problem



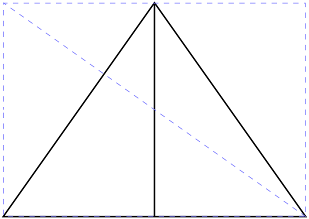
A shape $(L, \mathcal{S}_{\text{Michell},Q}, \lambda, t)$

Shape $\equiv L$: size $\quad + \quad \mathcal{S}$: scheme $\quad + \quad \lambda$: slenderness $\quad + \quad t$: thickness.

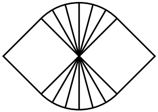
Galileo worked with all these parameters but scheme (he used the same scheme in each case). The idea of scheme was introduced by Aroca (ca. 1970).

The design theory: a short tour (Contemporary jargon, informal definitions)

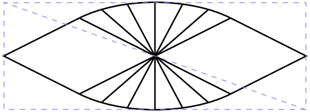
Structural Shape: a design view (no topology here)



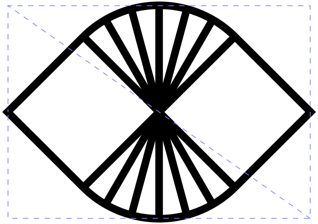
A shape with different scheme ($\mathcal{S}_{\text{Cuchillo español},Q}$) ...



...or different size ($L/2$)



...or different slenderness (2λ)



...or different thickness ($5t$)

The design theory: a short tour (Contemporary jargon, informal definitions)

Structural Shape: a design view (no topology here)

Remarks:

- The ‘‘thickness’’ property is a material density distribution —a member sizing law— in a general sense.
- The ‘‘constant thickness’’ property means a constant thick for each member of a Maxwell’s structure, not the same for all.
- In both cases, t stands for a scalar intensity, being constant the distribution or the proportions among members thicks.

Structural Shape: a design view (no topology here)

Strict sizing

	$L \to 0$	$L \to \mathcal{L}$
Net Strength	$t^2 \cdot f(\mathcal{S}, \lambda, L/\mathcal{L})$	
Stiffness	$t^2 \cdot f(\mathcal{S}, \lambda)$	?
Efficiency	$f(\mathcal{S}, \lambda, L/\mathcal{L})$	

Stability like stiffness?

Parkes's Hypothesis. (1965) The optimum structure for a Maxwell's problem is the stiffest among all other structures with equal maximum stress. ''The cheaper, the stiffest.'' ''Proof'' for a general deflection metric: strain energy account (''compliance requeriment'').

Improving structures

Theorem on optimum slenderness. (Aroca, ca. 1994)

The best slenderness for a scheme solving ''vertical forces problems'' is that for which the quantity of vertical structure is equal than that of horizontal one.

1. ''Give me a structural shape, I will return other that will be best but with equal scheme, thickness and size (maybe the same).''

$$\lambda_{\text{opt}}(\mathcal{S}) = \lambda\sqrt{\frac{\mathcal{V}_|}{\mathcal{V}_-}} \Rightarrow \mathcal{V}_{\text{opt}}(\mathcal{S}) \leq \mathcal{V}$$

Remarks: The theorem can be applied to any problem with parallel external forces. The slenderness and the quantities of structure has to be mesured in parallel and orthogonal directions to the loads.

Improving structures

Theorem on optimum slenderness. (Aroca, ca. 1994)

The best slenderness for a scheme solving ''vertical forces problems'' is that for which the quantity of vertical structure is equal than that of horizontal one.

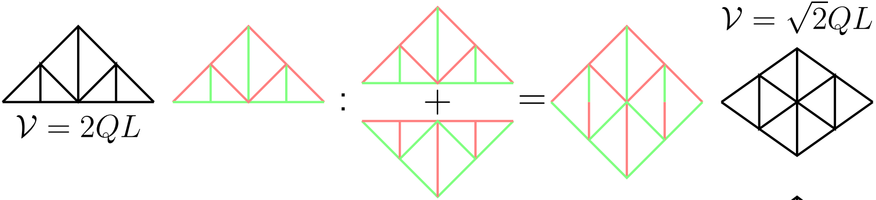
2. How does the quantity of structure grow with a non-optimum slenderness?

$$\mathcal{V} = \frac{1}{2}\mathcal{V}_{\mathrm{opt}}(\mathcal{S}) \cdot \left(\frac{\lambda_{\mathrm{opt}}(\mathcal{S})}{\lambda} + \frac{\lambda}{\lambda_{\mathrm{opt}}(\mathcal{S})} \right)$$

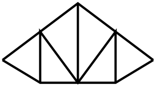
A very useful rule for everyday work!

Improving structures

Bending likes simetrical solutions (Theorem). If your structure is not simetric respect of the bend span and has optimal slenderness, make it simetric by the mean of a simple mirror, and multiply its original height by $\sqrt{2}$. It will look better now, won't it? (The quantity of structure is reduced as slenderness, by $\sqrt{2}$).



If you haven't freedom enough, try this as much as you can! →



The design theory: issues “to do”

- **Theoretical issues**
- **Semi-theoretical issues**
- **Structural Scope. Conjectures. Refutable, working hypothesis. Load cost in bending.**

The design theory: issues ‘‘to do’’

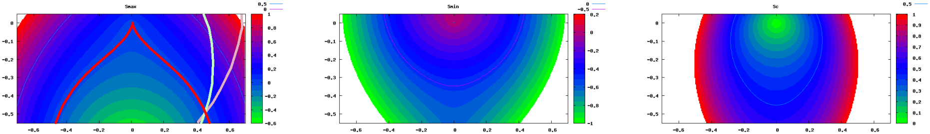
- **Theoretical issues**

  1. The Michell’s theorem gives us a sufficient condition. The main question is: for what kind of problems there is no layout that satisfies the optimality criterion?
  2. In problems for which the Michell criterion is not necessary, is there another?
  3. Can the Michell’s theorem be generalised in any way in order to include self-weight?
  4. Can (at least) an analogous theorem (and analogous PDE) be found for the pure self-weight case?
  5. Can the theorem of optimum slenderness be generalized for other problems that those of parallel forces?
  6. How big is the error of the Galileo’s rule in each Maxwell’s problem?

The design theory: issues "to do"

- **Semi-theoretical issues** (The analytical approach is so complicated that probably sooner than later one must recall in semi-numerical methods.)

   1. For a simple problem like vertical compression ¿could there be a solution with greater scope than the constant thickness solution? My own hypothesis is that the answer is **No**. But as I am unable of getting out a direct proof, I am looking for better solutions that may refuse my own thesis. If a systematic search over a fairly large set of compatible stress fields would fail to find a greater scope, the hypothesis on constant thickness will be harder.

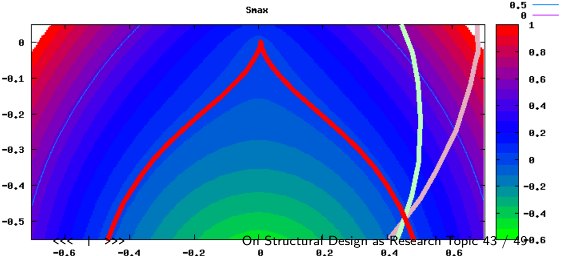


As it is customary in design theory, the method is the inverse than that of the analysis: in 2D, for each compatible stress field in equilibrium with self-weight, the shape of greater size is determined by three curves: $\sigma_a = 0$, $\sigma_b = -\mathbf{f}$, and $\sigma_c = \mathbf{f}$, and the scope of the field is the height.

The design theory: issues “to do”

- **Semi-theoretical issues** (The analytical approach is so complicated that probably sooner than later one must recall in semi-numerical methods.)

  1. For a simple problem like vertical compression ¿could there be a solution with greater scope than the constant thickness solution? My own hypothesis is that the answer is **No**. But as I am unable of getting out a direct proof, I am looking for better solutions that may refuse my own thesis. If a systematic search over a fairly large set of compatible stress fields would fail to find a greater scope, the hypothesis on constant thickness will be harder.
  2. Could we use FEM or other numerical, well-stated methods in the previous problem? With such a tool, we will be able to investigate the question for several failure criteria or for additional requirements, etc.



The design theory: issues ‘‘to do’’

- **Semi-theoretical issues** (The analytical approach is so complicated that probably sooner than later one must recall in semi-numerical methods.)

  3. «The state of the art about the bending problem is worst than that about the vertical compression problem: no Michell’s solution is known for $L \to 0$ (more precisely, I do not know it!).»

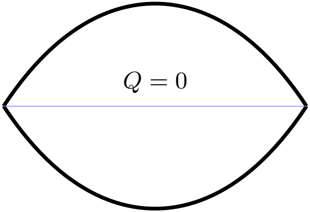
     (Wrote at March, 2011)

     a. My main concern here was to determine an optimal constant thickness solution only acting self-weight so the scope for the problem can be estimated. (2010)

$$L = \mathcal{L} = 1.325\mathcal{A}$$

$$\frac{\mathcal{V}}{(P+Q)\mathcal{L}} = 0.543$$

$$\frac{\mathcal{V} \div \mathcal{A}}{P} = 0.719$$



$\lambda_{\text{opt}} = 1.481$

The design theory: issues “to do”

- **Semi-theoretical issues** (The analytical approach is so complicated that probably sooner than later one must recall in semi-numerical methods.)

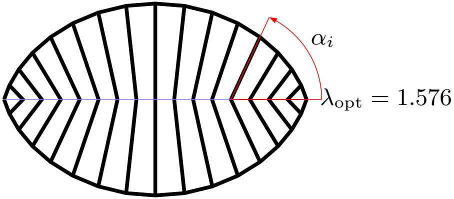
    3. «The state of the art about the bending problem is worst than that about the vertical compression problem: no Michell’s solution is known for $L \to 0$ (more precisely, I do not know it!).»

       (Wrote at March, 2011)

    b. Furthermore, I was able to determine a very good constant thickness solution only acting net load. This solution suggests strongly that the Michell’s theorem is not ruling the bridge problem. (Villamanta, 4/21/2011)

$$P \to 0 \quad (\mathcal{L} \approx 1.321\mathcal{A})$$

$$\frac{\mathcal{V}}{(P+Q)L} = 0.757$$

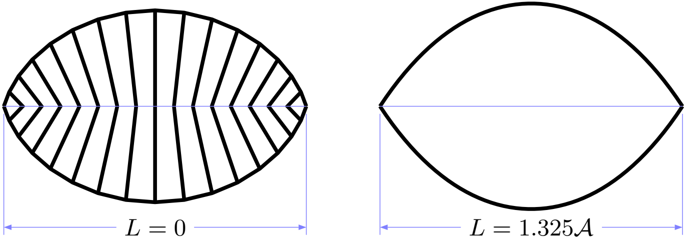


- **Semi-theoretical issues** (The analytical approach is so complicated that probably sooner than later one must recall in semi-numerical methods.)

3. «The state of the art about the bending problem is worst than that about the vertical compression problem: no Michell's solution is known for $L \to 0$ (more precisely, I do not know it!).»

   (Wrote at March, 2011)

   c. The transition between this two solutions is to be determined, as the accuracy of the Galileo's rule in this case.



The design theory: issues "to do"

Structural Scope. Conjectures.

Vertical compression problem: $\mathcal{L} = \mathcal{A}$, optimum solution: line with constant thickness (for normal steel ≈ 2.5 km with $\mathbf{f} = 200$ N/mm^2).

Bending problem:
From simulated annealing shapes, $\mathcal{L} \approx 1.23\mathcal{A}$ (for high steel, $\approx 7,7$ km with $\mathbf{f} = 500$ N/mm^2).
From best result up to date (catenary arc and cable), $\mathcal{L} \approx 1.33\mathcal{A}$ (for high steel, $\approx 8,6$ km with $\mathbf{f} = 500$ N/mm^2).

The design theory: issues "to do"

Structural Scope. Refutable, working hypothesis.

Main: For each class of Maxwell's problems of similar geometry, structural material and strength criterion, there is a structural size for which any structure with any shape colapses under the only action of its self-weight. No problem of greater size is solvable with such material and criterion.

Additional: The before mentioned size is the scope of the best structure for the problem, each of whose members has a constant thickness along its length.

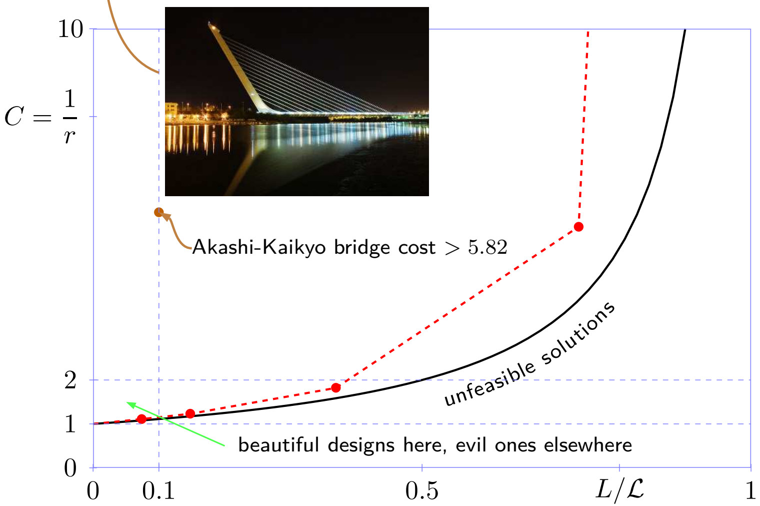
A refutation: (There are more...)

1. You should have to show a 2D shape that has a scope greater than $\mathbf{f} \div \rho$ for the vertical compression problem, with material $\mathbf{f}, \rho$, and elastic Von Mises failure criterion.

   This refuses the additional hypothesis.
2. If your shape has no limit over its feasible size at all, you have refused the main hypothesis too.

The design theory: issues "to do"

Load Cost (and relatives).



Further readings on this research: `http://habitat.aq.upm.es/gi/mve/dt/`

On Structural Design as Research Topic

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