Problem looks for solution.

Searching the "highest mountain".

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Contents

1 An idea since Galilei

Galilei [1638] considers a cylindrical column of height $\mathcal{L}$ that is just in ultimate equilibrium with its self-weight, i.e., $\mathcal{L}$ is the maximum height for the given shape and material strength. Any other cylindrical column of lesser height L could bear a useful weight over its head equal to $\boldsymbol{\rho}\mathbf{A}(\mathcal{L}-L)$, being $\boldsymbol{\rho}$ the specific weight and $\mathbf{A}$ the cross-section area, as with this weight the stress at the bottom will be equal in both cases.

If f is the maximum stress, $\boldsymbol{\rho}\mathbf{A}\mathcal{L} = f\mathbf{A}$, i.e., $\mathcal{L} = f \div \boldsymbol{\rho}$, being $f \div \boldsymbol{\rho}$ a characteristic length of the material. Let the *material scope* $\mathcal{A}$ be defined as $\mathcal{A} = f \div \boldsymbol{\rho}$. Let the *structure scope* $\mathcal{L}$ be defined as the maximum size for which a given structural scheme can bear its self-weight without any other load. For the cylindrical columns of Galilei we have simply $\mathcal{L} = \mathcal{A}$. (The general interest of the structure scope concept for the structural design theory is examined in Cervera Bravo [1990], Vázquez Espí [2011], Cervera and Vázquez [2011], Vázquez Espí [2012].)

How we can increase the structure scope? Galilei envisaged two ways: to increase the material scope —increasing f, or decreasing $\boldsymbol{\rho}$, or both—, or to change the structure shape. Let us consider the second one: is there any shape for the column problem such that its scope will be greater than $\mathcal{A}$? "For the column problem" means that the new shape can bear some useful load as the Galilei column do when its height is lesser than its scope.

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2 A general formulation of the problem

For the sake of brevity let us consider a 2D-universe. The body of given shape will be y-symmetrical, placed in the semi-plane $y > 0$ and supported in the $y = 0$ line. The displacement constraint will be $v(x, 0) = 0$, $u(0, y) = 0$. The material will be linear elastic subject to Von Mises criterion on stresses. There is not additional fundamental constraints on shape, but someone can be imposed for convenience (e.g., the maximum width can be fixed).

The Galilei column is simply a rectangular domain of height $\mathcal{A}$ and whatever width w. The principal stresses are $\sigma_{\mathrm{II}} = \sigma_y = -f(1 - y \div \mathcal{A})$ and $\sigma_{\mathrm{I}} = \sigma_x = 0$. Further, the Von Mises stress is $\sigma_{\mathrm{VM}} = f(1 - y \div \mathcal{A})$.

My working hypothesis is simply that there is not a shape that can have a height greater than $\mathcal{A}$. I have no proof albeit I got some evidences in favour of it as follows: I have analysed compatible and equilibrating stress fields —derived from complex potentials— determining the figure of maximum height fulfilling the above conditions for each field. And in all cases the height was strictly lesser than $\mathcal{A}$—excluding the field of the Galilei column.

If any one can envisaged a general proof of my hypothesis the problem would be directly solved. In any case else, the general formulation of the problem can be stated as to find out a such shape, proving in this way that my working hypothesis is false but being the shape of maximum scope determined. I am thinking this problem as a candidate for some topology or shape optimisation methods, see, e.g., Navarrina et al. [2005], Victoria et al. [2009], Zheng et al. [2009], París et al. [2010], Azegami et al. [2012].

Let me stress that traditional "constant stress bodies" [see, e.g., Keller, 1960, Karihaloo and Hemp, 1983, Cervera Bravo, 1990, Chai and Wang, 2005] are *pseudo*solutions for this problem, as the stress tensor is unbounded and the stress constraint (Von Mises) is not fulfilled. This is because in this kind of solutions we fix only σ_y as $-f$, but being the contour an exponential curve and a principal direction, the principal stress parallel to the contour will increase exponentially. [See Antuña and Vázquez Espí, 2012, for a detailed discussion].

3 Other related problems

As the general formulation can be hard to attack with available methods, I can suggest some alternative problems which in my view could be equivalent (or at least approximately equivalent) to the former.

Let be $V_0 = \mathcal{A}w$ a given volume in the 2D-universe. We can consider the problem of finding a shape with this given volume that maximise the height of the figure subject to the same stress tensor constraint.

Perhaps the stress constraint can be replaced by minimising the (maximum or mean) Von Mises stress in the volume, being the latter unbounded, and the total height of the figure fixed to a given value $\mathcal{A}$. With this problem it should be the case that we will get solutions with maximum absolute Von Mises stress lesser than f, hence with appropriated scaling we will get a solution higher than Galilei column.

Another approach arises from considering the calculus of the maximum scope shape as a limit case. Let us consider an useful load at a height $y = L > 0$ as an uniform load p along a width w_0. The problem is now to find a shape of minimal weight in equilibrium with p and its self-weight with the stress constraints as above. One additional constraint on the shape will be that it must lie into the region limited by $y \leq L$ and $y \geq 0$. If this problem can be solved, the structure scope $\mathcal{L}$ will be the limit of L when $p \to 0$ or $w_0 \to 0$. Obviously, a solution is a Galilei column of constant width equal to w_0, but is there another one? The useful load can be defined too as $P = \int_{-w_0/2}^{w_0/2} p(x)\,\mathrm{d}x$ being P a given constant. In this case the function p can be viewed as a design variable, or its integral over w_0 as a additional constraint on stresses.

More equivalents formulations can exist or can be proposed following these lines.

I think that a minimum compliance approach it is not equivalent to minimum weight one in this case due to self-weight. But it could be the case that minimum compliance objective leads to useful solutions after appropriate scaling providing the stress constraint is fulfilled.

4 Conclusion

The problem has theoretical interest for the structural design theory. It can be a benchmark problem for topology or shape optimisation methods. Each different stress constraint or material model (e.g., plasticity) lead to new instances of the problem.

I will appreciate any insight on it.

References

Joaquín Antuña and Mariano Vázquez Espí. ¿Existen problemas estructurales irresolubles? Una cuestión abierta. *Informes de la Construcción*, 64(525):103–109, 2012. doi: 10.3989/ic.10.061.

Hideyuki Azegami, Shota Fukumoto, and Taiki Aoyama. Shape optimization of continua using NURBS as basis functions. *Struct Multidisc Optim*, 45?, 2012. doi: 10.1007/s00158-012-0822-4.

J. Cervera and M. Vázquez. Galileo, Maxwell, Michell, Aroca: measuring the structural efficiency. In *Structural Milestone in Architecture and Engineering. International Conference on Research in Construction*, Madrid, november 2011. IETcc–UPM, Instituto de Ciencias de la Construcción (CSIC). ISBN 84-7292-401-7 84-7293-402-4. (Also available at `http://oa.upm.es/9931/`).

Jaime Cervera Bravo. Las estructuras y el peso propio. *Informes de la Construcción*, 42(407): 73–85, Junio 1990. URL `http://oa.upm.es/3830/`.

Y.H. Chai and C.M. Wang. Approximate Solution for the Shape of Submerged Funicular Arches with Self-weight. *Journal of Structural Engineering*, 131(3):399–404, 2005. doi: 10.1061/(ASCE)0733-9445(2005)131:3(399).

Galileo Galilei. *Discorsi e Dimostrazioni Matematiche*. Elsevierii, Leiden, 1638.

B.L. Karihaloo and W.S. Hemp. Maximum stregth/stiffness design of structural members in presence of self-weight. *Proc. R. Soc. Lond. A*, 389:119–132, 1983.

J.B. Keller. The shape of the strongest column. *Arch. ration. Mech. Analysis*, 5:275–285, 1960.

F. Navarrina, I. Muiños, I. Colominas, and M. Casteleiro. Topology optimization of structures: a minimum weight approach with stress constraints. *Advances in Engineering Software*, s.d.:s.d., 2005.

J. París, F. Navarrina, I. Colominas, and M. Casteleiro. Stress constraints sensitivity analysis in structural topology optimization. *Computer Methods in Applied Mechanics and Engineering*, 199:2110–2122, 2010.

Mariano Victoria, Pascual Martí, and Osvaldo M. Querin. Topology design of two-dimensional continuum structures using isolines. *Computers and Structures*, 87:101–109, 2009.

Mariano Vázquez Espí. On the structural design as a research topic. CIMNE Coffee Conference 2011, May 25th, Barcelona, May 2011. Also available at `http://habitat.aq.upm.es/gi/mve/dt/sad.pdf`.

Mariano Vázquez Espí. Measuring structural efficiency in bridges' sketches. CIMNE Coffee Conference 2012, Jan 18th, Barcelona, January 2012. Also available at `http://oa.upm.es/10401/`.

Bin Zheng, Ching jui Chang, and Hae Chang Gea. Topology optimization considering body forces. *International Journal for Simulation and Multidisciplinary Design Optimization*, 3:316–320, 2009. doi: 10.1051/ijsmdo:2009004.